Landamatics 关于教一般性人类思维的教学设计理论和方法

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这篇文章给我们的启发:

1. 启发我们如果做能力图谱分解，也就是挖掘出来针对具体问题的解决具体任务的完成的各个层次的知识。

2. 对任务和问题不断地做一般化，从而获得适用范围越来越大层次越来越高的知识，直到一般性人类思维。

3. 帮助人类如何提炼出来解决一类类问题的一般方法，进而“教会”大语言模型使得其看起来好像也会了这样的一般方法，从而这也是帮助人类学生学会一般方法的方法。

考虑到这篇文章写得非常不好读，又是用计算机科学家的语言在讨论最深刻的教和学什么以及如何教和学这个教育学的最核心的问题，我们重新排版[1]、翻译并且做了述评分享给大家。

[1]感谢王晓玲进行全文编辑排版，感谢张冠宇、赵崇臻绘图。

阅读指引

本书分 2 部分，第 1 部分是对原著的翻译和述评，第 2 部分是重新排版的原著原文。

第 1 部分包含全文翻译和对全文的 WHWM 述评。为了区分译文和述评，所有的述评都用了红色字体，并且用“述评”作为标识。

WHWM 述评包含以下 4 个问题：

1、What 作者说了什么？

2、How 作者是怎么说的？

3、Why 作者为什么要这样说？为什么说这个？

4、Meaningful 我觉得怎么样？

以上的 4 个问题，会在整篇文章、每个主题、段落、语句、字词的不同层次，嵌套使用。比如整篇文章的 What，要基于对文章中的各个主题追问 What 和关系，来获得。

目录

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第一部分

Landamatics，关于教一般性人类思维的教学设计理论和方法

述评：What 全文说了什么：兰达法（Landamatics）文章以直角三角形的识别为例介绍了其背后的思维过程，以及这样的思维的过程可以如何教给学生确保其能够掌握和应用这个思维过程。更进一步，文章从直角三角形拓展到了一般概念的识别，从概念识别的思维过程拓展到了命题判断的思维过程。通过这样的对某个任务背后的思维过程的分解和表达，以及不断地拓展到到这个任务背后的更加一般的任务，再到帮助学生通过内化和自动化来掌握这个思维过程，兰达法（Landamatics）文章展示了（不仅仅局限于命题判断的任务）一般性的思考的方法是可以教的，是可以帮助学生来掌握的，同时也展示了不断地一般化向着更高层次抽象的威力。

述评：How 怎么说的：下面展示了文章的每一节的主要内容。其中，我们发现，
- 摘要总结了全文内容
- 前言交代了全文的背景和目标
- 然后，2-3 两节指出当前教育的问题从而体现了兰达法（Landamatics）的

必要性
- 接着，整个第 4 节是对方法这个概念的说明（可以暂时忽略）
- 节 5 再次强调了传统教学中不教方法，从而更加体现了兰达法（Landamatics）的必要性
- 整个第 6 节呈现了命题判断的思考方法，从直角三角形的识别开始，到一般的概念的识别，再到命题的判断。在其中，也强调了不断地一步步更加一般化的过程，以及获得以及教和学这样的一般性的思考的方法的过程——分解和写下来、一步步操作、内化、自动化
- 整个第 7 节讨论了这样教一般性的思考的方法的好处，以及不这样做的原因和坏处
- 节 8 讨论了这个一般性的思考的方法到底有多么的一般
- 节 9 讨论了这个一般性的思考的方法和智力的关系
- 10、11 分别是交代了一下文章的局限，列出了文章的参考文献。

述评：Why 为什么说这个：作者希望以后的教学要围绕——教和学一般性的思考的方法，要采用分解和写下来、一步步操作、内化、自动化的方式来教，只有这样才能促进智力发展（智力本身不能教和学，但是促进智力提升的过程和方法可以教和学），提高教和学的效率。

述评：Why 为什么这样说：作者主要采用具体案例来做论述，其中层层一般化是其主要运用的逻辑结构，结合一些概念性思辨性的论述，主要通过例子以及例子的一般化来论述而不是主要依靠概念性思辨性论述使得这篇文章提供的方法更加具有可操作性。

述评：Meaningful 我觉得怎么样：这篇文章对我的启发很大。尽管实际上是倒过来的，我先概念化和系统化梳理和建构完我的体系，返回去看文献的时候，发现这篇最相关。

首先，在教和学什么上，不能仅仅停留在学科概念知识、事实性程序性知识的层面，需要走到关于获得和应用知识的知识的层面，而这个获得和应用知识的知识往往会体现为思考的方法。这正好和理解型学习的知识的层次之中的三四五层，也就是学科大图景、一般性人类思维、教和学的方法是一致的。当然，理解型学习进一步对这些知识做了具体化也是概念化的描述，使得其更具体更通用了。其次，在如何获得这个一般性的思考的方法上，对于一个任务或者一个问题做一步步操作的分解和明确写下来，是一个很好的方法。结合实验检验，也就是看一看分解出来的所有知识合起来是否就真的大多数时候解决了原来的问题完成了原来的任务，就可以得到完成相应任务或者解决相应问题所需要的各个层次的知识。在理解型学习这里，这个过程被称做“能力图谱分解”。兰达法和理解型学习在这一点上也是相通的。最后，在怎么教和学上，本文提出来进一步指导学生完成按照得到的思考的方法的指导语来做一步步的操作，然后，内化（扔掉指导语，自己来想着每一步）和自动化（不需要想每一步，就可以快速完成这个任务），也具有一定的参考价值。不过，这个教和学的方式的背后是“智力本身不能教和学，但是促进智力提升的过程和方法可以教和学”——这一点和理解型学习高度一致：不能上下左右贯通的高层知识是不可能被教和学的，这里的过程和方法其实就是把高层知识转化为流程性知识来教和学。

但是，回到具体内容，这篇文章的所谓方法 2-6，方法 2a-6a，及其整合起来的 7，逻辑上其实是有问题的，冗余的。原则上，对于判断命题（记作 $A \Rightarrow B$）的任务来说，我们只需要看命题的条件是否满足，满足则得到命题的结论，不满足则得不到命题的结论（得不到不表示命题的结论是不成立的，仅仅是得不到其成立这个结论）。无论条件 A 有多么的复杂，我们也只需要完成 A 自身的逻辑运算，我们根本就不用去按照 A 所包含的逻

辑运算及其顺序来对每一种类型做一个判断方法。因此，3-6 都是没有必要的，只需要 2。其次，把定义和双向都成立的命题当作一类把单向命题当作另一类分别做一个判断方法也是没有必要的：我们只需要知道定义是双向成立的命题就行了，然后，对 $A \Rightarrow B$ 和 $B \Rightarrow A$ 分别用一次判断方法就行。也就是说，2 也是没必要的，不过就是相当于两次 2a。因此，整个方法整合起来，我们只需要 2a。而且，2a 的关键非常非常地简单，可以写成下面这样。

判断一个命题 $A \Rightarrow B$ 是否成立的思考过程和方法如下：
- 识别出来条件 A，
- 对一个对象判断条件 A 是否满足，不需要考虑其他任何条件，
- 如果满足，则得到结论 B；如果不满足，则不能得到结论 B，或者说 B 是否成立不能判断。

回到直角三角形的例子，A 包含两个子判断以及它们的“和”关系——A_1（三角形）A_2（直角），B 就是“是一个直角三角形”。于是，对于一个给定的三角形，如果我们要判断其是否是直角三角形，我们完成
- 识别出来条件 $A = A_1A_2$，
- 对一个对象判断条件 A 是否满足，不需要考虑其他任何条件，也就是是否 A_1（三角形）和 A_2（直角）都满足，
- 如果 A 满足，则得到结论 B（是一个直角三角形）；如果不满足，则不能得到结论 B，或者说 B 是否成立不能判断。因为是定义，也就是 AB，也就是 $B \Rightarrow A$ 也成立，我们可以反过来再来一遍，
- 识别出来条件 B（是否是一个直角三角形），
- 对一个对象判断条件 B 是否满足，不需要考虑其他任何条件，也就是“是否是一个直角三角形”，
- 如果 B 满足，则得到结论 A（有一个直角同时是三角形）；如果不满足，则不能得到结论 A，或者说 A 是否成立不能判断。现在，把这两个方向上的判断“和”起来，就得到了

- 如果 A（有一个直角同时是三角形）满足，则得到结论 B（是一个直角三角形），
- 如果 B（是一个直角三角形）满足，则得到结论 A（有一个直角同时是三角形）；这个时候，如果条件 A 不成立，则 B 肯定不成立（这背后是反证法或者说命题和逆否命题等价。具体来说，可以这样论证：我们已经有前提 A 不成立，我们来看看 B 肯定不成立是否正确；我们来看看其反面——也就是 B 成立——是否成正确；根据前面半句，只要 B 成立，则 A 肯定成立，于是 B 成立肯定不正确，否则将导出 A 成立的结论，而这个结论和前提“A 不成立”矛盾），
- 于是，对于直角三角形的定义我们有“如果 A 成立，则 B 成立；如果 A 不成立，则 B 不成立”。

当然，这个逻辑上的冗余性问题，不影响文章的主要意思：
- 一般性的思考的方法值得和有必要教，可以教；
- 对于一个任务或者问题可以通过分解和写下来的方式来 **得到（发现）** 这样的一般性的思考的方法；（这里理解型学习拓展为能力图谱分解，并且要结合实验检验来看所分解出来的各个层次的知识是否完备和必要）
- 然后，通过一步步操作、内化和自动化的方式来帮助学生 **掌握**这个一般性的思考的方法，甚至可以带着学生自己 **发现**一遍；（这里是理解型学习高层知识具体化，先解决具体案例里面的问题）
- 对方法来源的具体问题做一般化，做抽象，不断地提升所解决的问题的通用性；（这里理解型学习称为上下贯通）
- 这个背后的理念是“过程和方法可以教，智慧不能直接教，只能通过过程和方法来教”。（这和理解型学习注重知识创造的过程以及过程背后的学科大图景等高层知识完全相通）

摘要

述评：简短总结了本文主要内容——用例子来展示兰达法，以及这样做的意义。

这篇文章展示了如何运用兰达法（Landamatics）来设计一个旨在教会人们一般性的思考的方法的过程。兰达法不是一堆课程计划，而是一个设计出来这样的能够高效地教会学习者的课程计划的一般性方法。这个方法给出了一个可以用于教任何知识和认知过程的一般流程。很多学生可以解决和他们被教的问题非常接近的问题，但是不能解决那些不太接近的问题。而这个根源在于，学生们并没有被教一个一般性的思考问题的方式，并且把这个教一般性的思考的方式当作一个教学的一般方法。和传统的教学方式中学生必须自己来形成这样的可迁移的一般化（generalization）来相比，兰达法帮助学生形成更加可靠的、科学的、概念性的可迁移的一般化。在本文中，这将通过帮助学生学会识别直角三角形的例子来展示。我们给教师们准备了一些策略。一种策略是引领学生们自己去发现概念和应用它。另一种是给学生所有的可能的关于这个概念的信息。第三种是把这两种方式相结合。教师选择什么策略应该根据教学的目标来设定，尽管第一种策略看起来更加有价值一些。兰达法的中心是帮助学生发现和意识到在概念定义的形成和概念运用中所需要的心智操作构成的系统，然后根据这些心智操作来制定相应的获得这些心智操作的教学系统。当然，在兰达法中，一定量的练习和运用对于内话这些心智操作也是必要的。这会一定程度上使得这些心智操作变成自动的。我们进一步把这个方法推广到适用于更加复杂的逻辑结构的概念的学习之中。最后，我们讨论了兰达法的优势。（4 张表，3 张图，11 篇参考文献。）

1　前言

述评： 交代背景和目标——用例子来展示兰达法，也就是“把一个问题解决过程背后的认知过程做分解-显性化而得到解决这个问题的所谓思考的方法，然后帮助学生内化-自动化这个方法，并且不断地把这个问题以及所得到的思考的方法做一般化”，讨论这样做的教育意义。

本文所描述的设计教和学思维方式思考方法的方法，之前被称作“算法式启发式理论”（Algorithmico-Heuristic Theory，简称 AHT）。它是一个关于行为、学习和教的方法。由于算法式启发式的教学系统以及其关注的心智过程可以用于一般性的思考过程，因此，AHT 实际上是一个关于一般性思维的教和学的理论和方法。

作者和他的合作者们关于 AHT 的第一个工作集中在知识运用场景中的思考过程和方法，也就是关于运用知识的研究。但是，从那以后，三件事情逐渐清晰起来。第一、之前关注的思考过程的方法不过就是更一般的认知活动的方法的特殊情况，而这些更一般的认知活动包含感知、记忆以及其他。第二，思考的方法也不仅仅局限于运用知识 i，还包含获得知识，以及更一般的学习知识。第三，前面发展起来的面向运用知识的教和学的方法也使得获取知识的过程更加高效。这个原因很好理解：人们往往不是通过听和读知识来学习知识的（除非，学习的目标仅仅是了解性的），人们往往通过运用知识来学习知识——这也是练习的主要目的。因此，很显然一个面向知识运用的好的教和学的方法也会是一个知识获得的好方法：只要设计合理，一个知识的运用过程往往是获得知识的过程的一个重要组成部分。

在第一个工作完成之后，我们还进一步看到这个教认知活动的方法不仅仅影响学习和思考的过程，还进一步影响了某些思维的和个性的特质的形成，例如智力、直觉、自我管理和控制、思维的组织，以及对于学习能力和解决问题能力的更高层次的自信，甚至包含在面对问题、构建解决问题的策略、在实际着手实现解决方案之前分析问题等等这样的系统性特质。

简短地说，AHT 的适用范围大大超过了原始的“算法式启发式理论”名称所涵盖的。AHT 实际上成了一个关于行为、学习和教的一般性的理论。因此，再看到了这个 AHT 的更一般的适用范围和其名称所含的适用范围的差异之后，纽约城市

大学（CUNY）的 Berkowitz 就给 AHT 取了一个新名字“兰达法（Landamatics)”。一开始我们不太接受和使用这个名称。但是，因为这个名称被用的越来越广泛了(甚至在互联网上也是如此)，在这里我们就也用这个名称。

兰达法不是一个关于具体的一个或者两个学科、概念或者技能的教学法。它也不是一堆课程计划。反而，它是一个设计任何一个高效率的教学计划的方法，无论这个教的内容是某些现象、对某个对象的视觉分析的过程，或者思维的策略等等。这个方法会给出来通用的又是具有针对性的足够具体的细节的设计和教任何一个领域的具体知识以及认知过程的算法式的或者费算法式的过程。

在这一章中，我们会展示如何来设计这样一个运用兰达法的理论和方法来教和学一般思考方法的教学过程。

在所有的可以运用兰达法来教的认知过程的方法之中，在这一章中，作为展示，我们选择了教思考的方法。在所有的不同的思考（例如解释、证明、总结归纳）之中，我们选择了识别这个思考任务，也就是基于这个类别的定义判断某个对象是否属于某个类别。在本章后面的部分，这个任务会被拓展到更一般的情形，而不仅仅是做出来是否属于某个类别的结论，还会做出来关于这些对象的属性、对象之间的关系的结论。这些结论往往需要用到超过定义的其他的命题，例如自然规律、公理、定理、规则等等。

总结一下，我们说，本章的目的是，通过所选择的案例，来阐述一个教思考方法的一般方法——兰达法，同时展示教什么和怎么教。注意，为了达到这个目的，教什么和怎么教同样重要。

下面是关于我们所用的名词得一点说明。**任何一个思考过程我们都看作是一个运用某知识的过程**。思考过程的不同首先就是运用知识的目的不同：可能是为了识别事物、解释事物、证明事物（命题）等等。这个目的和问题的条件合起来，就决定了（需要用到什么知识，译者添加，以及）所要用到的知识应该如何被使用，同时大脑需要开展那些认知活动，才能达成这个目标。尽管人们经常遇到“视觉思考”这样的表达，在这里，“思考”更多地是更一般的含义，而不仅仅是这个字面含义。思考，和直觉和想像相反，往往其加工对象是概念和命题，而不是图象，尽管图象也往往会和思考过程相联系。运用知识的含义，原则上比运用概念和命题要广泛，因为其也可以包含运用图象。但是，在本文中，当我们说“运用知识的方法”的时候，我们的知识往往指的是概念和命题。

先在我们来看下面两句话“一个运用知识来做对象识别的方法”和“一个做对

象识别的方法”。这两句话表达了相同的含义，因为识别过程就是需要用到相应的知识来完成的。往后，我们不再区分这两句话。同时，由于在本章中，我们选择对象识别任务仅仅当作展示我们的一般性的使用知识的方法的案例，我们往往会用含义更广泛的“一个使用知识的方法”而不是“一个使用知识来完成对象识别的方法”。这意味着我们关于对象识别方法的论断是可以迁移到其他的思考任务甚至所有的认知活动之上的。

2 教会一般性的认知过程和思考的方法当作教育最重要的目标之一

述评：教会一般性的认知过程和思考的方法当作教育最重要的目标之一：为什么要教一般性的思考方法：具体知识变化太快，这些一般性的思考方法变化慢，通用性强。

在关于教育目标和目的的讨论中反复出现的一个事实是，在现代以信息为基础的工业社会中，新知识的发展速度非常快。知识变化如此之快，以至于我们今天所学的内容可能在十年后，甚至可能在几年后就会过时。(当然，人们也要意识到，并不是所有的知识都在随着时间概念，都会变得过时。例如，关于数学和力学的规律、关于历史事实和事件、关于地理现象等等的基础知识是非常稳定的，甚至永恒的。)

于是我们就有了下面的问题：如果在科学和技术的发展过程中，知识是经常在改变的，那么，获得和运用知识的认知活动机制，在这个过程中，是否也是在改变的呢？更准确地说，它们的改变和人类所获取的知识本身的改变一样快吗？答案是否定的。

那些在相应的领域中已经掌握了如何高效率地去获取和运用知识的无论在哪个科学、技术或者实践领域的专家，往往在使用着想用的认知操作和过程（从一个这样的武器库里面来）来学习和运用不同的知识。这些过程可能对于不同种类的知识是不同的（例如，事实性知识和关于自然规律的知识），对于不同种类的问题也是不同的。但是，只要知识的种类和问题的种类相同，这些过程就是相同的。

因此，所获得和应用的知识可以不同，但是，获得和应用的方法是相同的。因此所用的获得和应用知识的过程和机制相同，我们说，在这个意义上，机制是内容无关的，通用的。

如果我们接受这个论点"学会获得和应用知识——任何的知识、不同的知识——和学会具体的基础知识同样地重要（或者，我们应该说前者更加重要，因为任何具体知识都有可能不久就会过时）"，那么，教会学生一般性的认知过程和相应的方法就是教育的关键目标之一。

当然，教会适用于一般知识的认知过程和方法只有通过学习具体领域的知识来实现。然后，进一步的问题在于：是否学习具体知识本身也可以是一个教育的目标，还是仅仅当作学到一般性的认知过程和方法的方式。

3　教育中的奇怪现象

述评： 当前的教学一般不教一般性的思考方法，这产生了问题——具体知识可能学到了但是学习、创造和运用知识的知识没学到

在学校教育中，经常有一个奇怪的现象：学生们往往被要求去识别对象、解释事物、作出结论、证明命题等等等等，但是他们往往没有被教过——也不知道——什么是识别、什么是解释、作出结论和证明命题意味着要做什么。问题在于：并不是说这些过程没有形式上的定义（仅仅给出定义不解决什么问题，不教学生太多），而是没有教给学生关于这些过程中所需要进行的相应的心智操作。比如说，一位老师说，"做解释就是使得某个东西更清楚"这没有任何意义。反过来，如果我们说清楚了，我们在认知中应该完成哪些事情才能使得一个解释更加清楚，那就是有意义的。

我们开展的对教师的大量的访谈都说明，在绝大多数情况下，教师们不知道如果要完成其所教和所要求学生完成的任务中有哪些心智操作。换句话说，教师们自己也没有完成这些任务的合适的方法，因为也没人教过他们这些啊。

因此，这一点也不奇怪：根据《纽约时报》报道（见 The New York Times, 1997 年五月 4 日），最近的美国国家教育进展评估发现"根据最近的科学教育美国国家检测结果，美国学生对于基本的科学事实和原理有一定的理解，但是他们在

运用科学知识、设计实验或者清楚地解释其背后的逻辑上的能力令人失望”。

4　“方法”一词在科学和日常生活中的两种含义

述评： 补充说明这里的“方法”指的是什么，区分了行为（M_a）和处方（M_p）。

在哲学和科学的文献中存在着许多个“方法”的定义。它们往往造成了误解，影响了甚至科学家之间的沟通。为了解决这个问题，我们先来看看在日常语言中“方法”的含义。

基于语言使用的语义分析告诉我们“方法”有两种含义两种使用场景：(a) 解决某个问题或者完成某个任务的行为（例如“科学家们发现了诊断某个疾病的方法”），(b) 指出来要做哪些行为才能完成任务或者解决问题的指导语或者说处方（例如“一个数学家提出了一种解决某一类问题的方法”）。

为了区分“方法”的这两种不同的但是相互联系的含义，我们用 M_a 来表示行为，用 M_p 来表示处方。我们将用这些符号来表示从上下文中不容易区分出来到底指的是哪一种的情形。对于可以从上下文区分的情形，或者同时指代了两者的情形，我们将不做这样的区分。

一般来说，当人们寻找解决新问题或者完成新任务的方法的时候，先需要发现（找到）这些行为，然后用语言描述或者表示出来这些行为，于是也就成了处方。换句话说，人们往往先发现 M_a 然后把它们转化为 M_p。

4.1　准确地说，什么是“方法”

述评： 进一步补充说明这里的“方法”指的是什么，有目标有整体有行为步骤。

我们把“方法”定义为为了实现某个目标的一个有结构的指导语系统或者行为系统。

这个定义抓住了方法的下列重要特征：

1. 一个方法总是一系列的指导语或者行为，或者说指导语或者行为的一个系统。只有在最极端的情况下，这个系统只包含一个单独的指导语或者行为。

2. 一个方法总是一个有结构的对象，其背后是一系列基本的元素（指导语或者行为），用某种方式连接联系起来（例如，按照某个流程或者层次性结构来相连）。

3. 一个方法总是面向某个目标的，例如为了解决某个问题为了完成某个任务。这也是人们经常用“为了... 的方法”来称呼一个方法的原因，尽管有的时候也用“关于... 的方法”。

在日常语言和科学中，方法这个词往往和一些典型词汇像联系，例如“过程”、“流程”、“指南”、“技术”、“策略”等等。当然，其中的某些词，例如“策略”，往往比“方法”还模糊和多义。

4.2　方法作为客观的社会现象和主观的心理现象

述评：进一步讨论这里的“方法”是什么。

当概念和命题在社会实践和科学中被发展出来以后，它们会在语言的世界中客观化（物质化）。一旦客观化了，它们就会成为一种社会现象，例如以出版的或者电子的形式存在供人们学习和使用。当人们学习到它们之后，它们就成了学习者的主观心理现象，对应着那个客观社会现象的主观心理现象。

主观方法可能遵从也可能不遵从客观方法。例如，很多人给出的关于事物的解释往往是错误的（不正确的、迷信的、不一致的等等）。他们主观解释的方法可能不符合科学中发展出的有效的客观化解释方法。

因此，教育的一个重要目标就是，这样来教方法使得学生认知结构中形成的主观方法和在社会实践和科学中形成的客观化的方法相一致。

4.3　方法和技能的关系

述评：进一步讨论这里的“方法”是什么：方法和技能的关系。

尽量两者有联系，但是方法和技能并不相同，因为其反映了不同的心理现象。

以 M_p 为例，显然，一个关于要完成哪些行为才能达成目标的处方和实际执行这些行为是不同的。例如，一个人可以很清楚如何游泳（每一步做什么就会游了），但是不会真的游泳（不会真的游起来）。在这个意义上，方法（M_p）和技能是非常不同的。

同样，我们也需要区分方法 M_a 和技能。技能不是行为系统。技能是大脑中的代表了完成行为系统的生理过程，也就是执行 M_a 的潜在的能力。举例来说更清楚：当有人说一个外科医生技能高超的时候，往往并不是说这个医生正在完成某个手术执行某些行为，反过来，这意味着这个医生具有完成这些行为的潜力，他可以在需要的时候执行这些行为。因此，方法作为行动的系统 M_a，以及作为处方 M_p，和技能都不同。

当然，在构成 M_a 的行为和技能之间有直接的联系：行为的执行会导致大脑中生理过程的形成——行为执行结束以后留下的痕迹会帮助形成技能。也就是说，一方面，技能是行为执行的结果；另一方面，技能代表了未来执行行为的可能性。

或者说，技能是潜在的行为（或者说行为系统）。

注意到技能只有通过执行行为来形成，我们说，为了帮助学生形成好的高效的技能，必须教给学生好的方法。教授方法是形成技能的一种方式。

4.4　关于方法的知识和操作的区别

述评：进一步讨论这里的“方法”是什么：掌握方法的知识和掌握方法的操作两者之间的区别。

我们需要对掌握方法的知识和掌握方法的操作作出明确的区分。指导一个方法的知识，意味着知道了这个方法的指导语，也就是可以用语言的方式把这个指导语描述出来。掌握方法的操作意味着可以完成相应的物理的或者心智的行为操作。在学校和现实生活中，下面的情况都可以能发生：

1. 一个人掌握了方法的知识，也掌握了方法的操作。
2. 一个人掌握了方法的知识，但是不掌握方法的操作

例如：一个人知道怎么游泳的知识（掌握了需要做哪些行为就会游泳了的知

识）但是不会真的游起来。

3. 一个人不掌握方法的知识，但是掌握了方法的操作。

例如：一个人会解决按照某些特定的步骤（也就是会了 M_a）来解决某一类问题，但是描述不出来相应的指导语

4. 一个人既没有掌握方法的知识，也没有掌握方法的操作。

5　传统教学中不能教会学生思考的方法及其恶果

述评：传统教学不教方法的恶果。

一个所有教师都经常遇到的问题是：很多学生可以解决和所学习到的问题非常相似的问题，但是不能解决那些实际上属于同一种类型的但是其看来没有这么大的相似性的问题。学生们在这样的问题中经常出错。为什么？

我们的分析表明，这是因为学生们被教了求解过程，而且这个求解过程的呈现方式也导致其只能处理几乎完全相同的问题，而没有被教思考和推理的方法——当作解决一个足够通用的问题的心智过程的指导语来教。

这样的教的方式是典型的被广泛采用的。因此，作为这样的教的结果，行为系统 M_a 仅仅和学生被教的时候所用的具体内容建立了联系。于是，未来也只能在解决足够相似的问题的时候这个操作会被唤起。这就是为什么，当学生面对实际上属于同一类型但是表面看起来不那么相似的问题的时候，他们要么不知道如何下手，要么出错。

6　传统教学中所用方法的一个例子

述评：传统教学不教方法的一个例子——以直角三角形的识别为例，前置概念知识清楚的学生仍然用非定义中出现的特征来判断三角形而出错。

我们的在莫斯科一般和教育心理学的前同事 Zykova 博士展示了一节来自于莫

斯科的学校的几何学课。这节课的主题是“直角三角形”。这是老师和学生在课堂中做的事情：

教学行为 *1.* 老师解释了三角形有几种不同的类型，每一种都有其自己的特征。今天，他们将学习其中的一种——直角三角形，并给出了定义（“有一个角是直角，也就是 90° 的三角形是直角三角形”）。

教学行为 *2.* 教师展示了几个直角三角形的例子。

教学行为 *3.* 然后，教师给学生提供了一些练习题来帮助学生学会直角三角形的定义以及学会如何使用这个定义：

行为 3a. 教师让学生重复直角三角形的定义。一部分学生完成了这个任务。

行为 3b. 教师让学生在黑板上画出来几个直角三角形的例子。两位学生做对了。

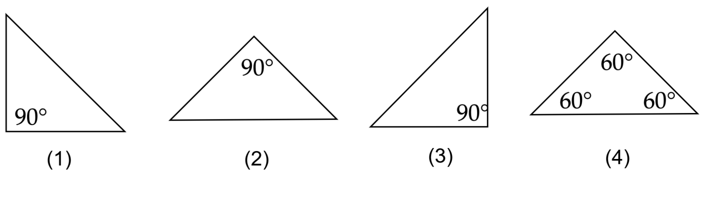
行为 3c. 接着，她给学生在黑板上展示了几个三角形，其中有直角三角形，也有非直角三角形，然后，她让学生把直角三角形识别出来。几位学生做对了。

行为 3d. 最后，她问学生是否还有问题，是否都清楚了。学生们异口同声地积极地回答道没有问题了，都很清楚了。

一切都很顺利，学生和老师都确信学生们掌握了直角三角形的概念，也知道如何去使用它了。

课后，Zykova 博士问其中的一位学生——这位学生在课堂中表现非常积极老师提出的所有问题也都会大正确了——来做了下面的小实验。在这个实验中，Zykova 博士首先让学生给出来直角三角形的定义，学生给出了正确的回答。

然后，她给学生下面的四个三角形，请学生识别哪一些是直角三角形：



学生选择了 (1) 和 (3)。

主试：为什么没选 *(2)*？它有一个直角。它难道不是一个直角三角形吗？

学生：不是，因为直角三角形要求直角必须在三角形的底边上，底边的左侧或

者右侧。

我们看到，尽管学生正确地给出了直角三角形的定义，这个定义不过就是教师在定义直角三角形的时候所说的话的机械重复。这位学生对于直角三角形的概念的认识实际上和教师给出并且学生准确地重复的定义是不同的。在正确的定义中，完全没有关于直角的未知的任何特征。但是，学生所接受到的定义包含了这样的无关特征，因此，其概念是错的（更加狭窄的）。

那学生的错误的概念是如何在其认知结构中形成的呢，尤其考虑到教师的教是非常正常地实行的（有概念定义、正面和反面的例子，有练习，等等）？尽管学生知道和正确地给出了直角三角形的定义，但是，他实际上的直角三角形的概念来自于对于其在学习这个概念的时候遇到的所有的直角三角形的经验拓展一般化，而正好，在这个具体的课堂中，这位学生所遇到的所有的直角三角形的具体例子包含了直角的位置的信息——他把这个信息也做了经验拓展一般化。

Zykova 进一步论断，这位学生所犯的错误，主要来自于学习直角三角形概念的时候所遇到的例子太有限，也就是“有限变体”的问题。这些学生都只遇到了直角在底边的左右侧的直角三角形（也就是 (1) 和 (3)）。于是，Zykova(Zykova, 1963) 以及其他有意无意推崇变式教学和经验拓展一般化的人给出的教学上的解决方案就是：教师在介绍新概念的时候要提供足够多的变式，多到把所有的无关特征都覆盖，都让学生看到这些无关特征不能算定义里面包含的特征。这样就可以防止学生做有限一般化而得到更加狭窄的概念了。

是的，包含了无关特征的有限的变式确实是学生形成错误概念，作出错误判断和解答的原因之一。但是，我们认为，真正的原因不在这里。

6.1　为什么不可能基于经验拓展一般化来提供高质量的教和学

述评：帮助学生降低出错率的变式教学的问题，变式总赶不上需要考虑的非定义特征多。

经验拓展一般化，通过变化不相关的对像特征来帮助学习者自己总结出来一般概念，在无关变量不多变量取值也非常少的情况下是好用的。但是，当变量数量和变量取值数量超过“非常少”的时候，所需要的变式的数量就太多了，以至于在

实际教学过程中就不可能提供所有的这样的变式。例如，当无关变量是颜色、大小、空间方向这三个，每一个变量只有几个取值的时候，合起来需要的变式就会达到十多个。就算仅仅由于时间的原因，也没有老师可以做到对同一个对象给出来十几个这样的变式。

反过来，如果所提供的变式的数量比客观需要达成概念理解的要少，则总有有一定的可能当学生来做这样的经验拓展一般化的时候得到的“概念”和这个概念本身的内涵不一致。在今天的学校教育中，这样的学生认知结构中的概念和这个概念的科学的含义之间的差异其实是经常发生的。

所提供的变式的数量比达成对概念理解所需要的变式的数量少的越多，则就会有更大的可能（a）在学生认知结构中所形成的一般化是不构的，并且因此（b）基于这个一般化所形成的对概念的理解也往往是不准确的。

根据兰达法，这样的概念形成和概念应用的错误的真正的根源是学生没有被教概念获得和概念应用的一般方法——这种方法不需要穷尽实践上不可行的所有变式。如果这个判断是正确的，那么，这个问题的解药也就不应该是穷尽所有可能的变式，而是教给学生一般性的思考的方法。这才是超越经验拓展一般化的局限，帮助学生达到对所学习的概念的理解和这个概念在科学上的含义完全一致的境界的方法。

6.2 超越经验拓展一般化的局限来传递概念和认知操作的兰达法——通过教给学生思考的方法

述评：帮助学生降低出错率的真正方案——通过教给学生通过概念来识别对象的一般性的思考的方法——的好处。

兰达法发展和推崇一个完全不同的方法来完成一般化，形成概念和完成思考过程，通过有目的性很强的显式地教给学生思考的方法（包括 M_p 和 M_a）。

这个方法：

- 使得大量的变式变得没有必要，
- 保证形成正确的足够的一般化，
- 保证形成正确的概念和命题，在足够的一般化的基础上，

- 保证学生形成高效的获取和运用知识（图象、概念和命题）的方法，

- 保证最通用的和最准确的传递知识和心智操作，而不仅仅是经验，到新的问题和场景之中去，

- 保证大大降低学习中的错误和困难，

- 保证学习者发展对心智操作的自我管理、自我控制、自我调控的能力，

- 使得上面的所有目标都可以实现，并且是可靠地和相对快速地实现。

和传统教学的经验拓展一般化相比，兰达法帮助学生在认知结构中形成更加 **可靠的，符合概念的科学含义的（reliable, scientific concept- congruous, RSCC）**一般化。

6.3　基于兰达法教思考方法原理的教学设计

述评：兰达法——通过教给学生通过概念来识别对象的一般性的思考的方法——的整体流程。

为了做一个简单的对比，下面我们仍然用教会学生直角三角形的概念并且把这个概念用于完成识别直角三角形的任务为例，来展示经验拓展一般化和可靠的符合概念的科学含义的（RSCC）基于兰达法的一般化之间的差异。

和教其他任何东西一样，有两种基本方式（策略、方法）来教直角三角形的概念：(1) 让学生自己做出来什么是直角三角形的发现，在教师的指导下；(2) 教给学生作为结果的知识以及相应的运用知识的方法，跳过发现这个知识的过程。

我们将从地一种策略（方法）开始。下面是这个策略的教学目标和活动：

1. 学生独立发现直角三角形的概念。
2. 想出来这样的三角形的名称（用学科概念中通用的指代这个概念的名词）。
3. 把这个概念的定义正确地有逻辑地表达出来。
4. 学生独立地发现运用这个概念的心智操作（M_a）。
5. 把这个心智操作明确表达出来（M_p）。
6. 学习和练习如何运用总结出来的上面的两个方法。
7. 内化这个方法（M_p）。
8. 自动化这个方法（M_a），从而保证真正掌握这个概念的知识和应用。

6.3.1　策略（方法）1

述评：兰达法的具体案例——以直角三角形的识别为例，学生在教师指导下发现和分解出来每一步，明确写出来，内化，自动化。

（指导学生独立发现一个概念，其名称或者符号，其定义，以及应用这个定义的方法）

在上面列出的靶向目标和活动中，我们将详细描述其中的最后五项。这五项正好可以体现兰达法。

教学目标 4：让学生发现和有意识地认识到运用学会的概念及其定义来完成识别这个概念——在这里就是识别出来哪些是直角三角形——所需要的心智操作（M_a）。

教学行为：

1. 问学生，基于直角三角形的定义，在他们的大脑中得有什么才能完成这个判断一个三角形是否是直角三角形的任务。

学生们回答他们需要去检验这个三角形是否具有一个直角。

教学目标 5: 让学生明确地表达出来相应的指导语（M_p）;

教学行为：

1. 让学生详细地表达出来，对于一个还不能用直角三角形的定义来识别直角三角形的人来说，需要在其大脑中完成哪些事情才能够判断一个三角形是否是直角三角形。

2. 如果学生可以把这样的方法正确地写出来，则进入到下一步；否则，给学生进一步解释（由于空间所限，解释的细节没有在这里提供）怎样来表述一个识别直角三角形的方法（M_p）。

在教师的帮助下如果需要的话，学生表达下面的方法：

1. 从直角三角形的定义中把最重要的特征分离出来——一个 90° 的角。

2. 在大脑中在三角形的基础上来运用这个特征——检查一下是否有一个角是 90° 的。

3. 按照下面的规则做出来结论：1. 如果一个三角形有一个 90° 的角，则这个三角形就是直角三角形。

2. 如果一个三角形没有一个 90° 的角，则这个三角形就不是直角三角形。

4. 在黑板上把这个方法（算法、流程）写下来，或者写在其他的介质上（如果提前准备过的话）。

教学目标 6: 提供练习这个方法（M_p）的例子.

教学行为:

1. 告诉学生当前的任务是把前面表述清楚的方法用来识别直角三角形。

2. 给学生不同的三角形，让他们用前面的方法来识别哪些是哪些不是直角三角形。

3. 给学生解释一下应该一步步地来使用前面的办法：先看第一步指导语，按照指导语完成相应的心智操作，然后看第二步完成相应的操作，等等等等。

按照之前给出的方法，学生应该很容易就能够识别出来哪些是直角三角形，无论直角的位置在哪里。

教学目标 7: 给学生提供进一步的练习的例子，来帮助学生实现这个方法的内化和完全掌握。

教学行为:

1. 告诉学生，他们看起来好像可以不依靠写在黑板上的指导语来完成这个任务，而是自己来指导自己（自己给自己下命令，每一步做什么）。

2. 告诉学生，从现在起，你将从把前面的方法从被搬上擦掉，学生们将不能按照前面写下来的方法来一步步地操作，而是只能自己给自己下每一步做什么才能识别直角三角形的指导语。

学生们应该能够做到通过他们自己给自己下指导语来完成相应的识别直角三角形所需要的心智操作（M_a）。

教学目标 8: 实现心智活动（M_a）的自动化。

教学行为:

1. 告诉学生他们看起来似乎也不再需要自己给自己下指导语了，他们先在完全可以自动化地跳过指导语来完成识别直角三角形的任务了。

2. 给学生最后一批三角形，要求学生识别出来其中的直角三角形。要求学生尽可能快速地作出判断，不再需要借助自己下指导语这一步。

学生应该能够轻松而迅速地完成这个识别直角三角形的任务了。

这就完成了兰达法教学设计的策略 1。

尽管我们这里对用兰达法来教和学直角三角形的概念以及运用这个概念来识

别三角形的描述比较长，但是实际上，整个过程也就 15-20 分钟。

6.3.1.1 — 关于兰达法内化和自动化的心理机制的说明

述评：对兰达法的补充说明，尤其是内化和自动化的心理机制。

什么是把方法（M_p）内化和把方法（M_a）自动化？在内化和自动化的过程中，人的认知结构中都发生了什么？

根据兰达法的理论，一个方法的逐渐内化和自动化不过就是学习和练习中的一个逐渐的转变，从一种执行机制到另一种执行机制。

1. 在学习一个方法的第一阶段，操作是根据外部的（打印的或者电子的）指导语来进行的。

2. 在第二阶段，操作是有学生内部自己发起的，但是仍然需要学生自己先给出来每一步做什么的指导语。这一步实际上是方法（M_p）的内化。

3. 在第三阶段，学生再也不需要外部的或者内部的指导语了，学生只需要从任务的目标和当前的条件出发就可以来完成这个任务。这一步实际上是方法（M_a）的自动化。

在从一个阶段到另一个阶段的转变的过程中，心智过程对应着的心理机制，根据兰达法，也经过了一个关键的转变：在第一和第二阶段，操作的执行是序贯性的（一步一步的），在第三阶段，心智操作开始并行或者部分并行。

心智操作的并行使得下面成为可能：

- 并行处理信息，而不是序贯性地一步一步地处理信息，
- 把对象当作整体或者模式识别出来，
- 非常迅速地几乎就是瞬时地完成心智操作，
- 几乎不费劲地完成心智操作（过程），就好像它们是自己完成的一样。

这些特征意味着对方法的完整的掌握和自动化。

在传统教学中，这些特征是自发地无计划地低效率地偶然形成的，如果还能形成的话。兰达法使得这个形成过程成了一个可以有计划地有目标地开展的过程，因此，也就是保证了这样的高质量的心智能力成了方法的自动化并行性的一个自然的结果。

6.3.2 策略（方法）2

述评：兰达法的具体案例——以直角三角形的识别为例，教师已经完成发现和分解，也明确写出来了，学生去做内化和自动化。

（用概念、名词、定义、方法的现有的形式，结果的形式来教，而不是通过学生自己发现的方式）

在策略 2 中，和之前的策略 1 中学生自己来发现直角三角形的概念自己来表述其定义不同，教师把所有这些知识都当作已经创造好的知识来教给学生（配上适当的展示和练习）[1]

6.3.3 选择策略 1 或者 2 的条件

述评：策略 1 和策略 2 的选择。

显然，在教育上，策略 1 是比策略 2 更加有价值，有好处的方法。但是策略 1 更耗时。

因此，看起来，决定选择这两种策略的哪一种的唯一的条件就是时间。当然，往往我们可能没有时间来实现完整版的策略 1，但是又有超过了策略 2 所需要的事件。这个时候，兰达法建议采用两种策略的某种混合。我们把这个混合或者说组合方法成为策略 3。

6.3.4 策略（方法）3

述评：策略 3——策略 1 和策略 2 的融合。

（组合策略）

[1]这里方法（M_a 和 M_p）本身也不是学生通过完成任务然后开展思考而自己构建起来的，而是教师已经表述好的，直接教给学生，然后学生来通过练习完成内化和自动化的。

在这个策略中，这个概念的某些方面是通过发现策略来教的，某些方面是通过知识结果的方式来教的。到底哪些知识通过什么策略来教，完全由教师根据当时的教学目标、对各个策略的优缺点的比较来决定的。

6.4　上述几何课案例的主要缺点

述评： 提出问题：这些方法可以一般化成为识别一般概念的方法吗?。

上面表述出来的识别直角三角形的方法可以用于识别所有的直角三角形。在这一点上，它是通用的。但是，同时它又太具体了——它仅仅可以用于识别直角三角形。

它可以被拓展为可以用于其他场合吗？可以把这个方法推广一下吗？答案是“可以”。

6.5　提升方法的一般性

述评： 回答问题：从识别直角三角形一般化成为识别一般概念的方法，给出方法 2。

作为一个例子和起点，我们还是以之前就表述过的识别直角三角形的方法为例。我们把这个具体的通用程度低的方法记作 d_1，我们把下一级一般化的方法记作 d_2，以此类推。在这里，由于识别直角三角形的通用程度最低，因此就被记作 d_1。在我们后面的表格里面，一个更低通用度的方法将被放在表格的左侧。其后侧将会放一个通用程度更高的方法。这样的布局将会使得我们关于不同通用程度的方法的比较更容易进行。不同方法之间的差异将会用特殊字体标注出来。

表 1 通用性拓展 1：从 d_1 到 d_2

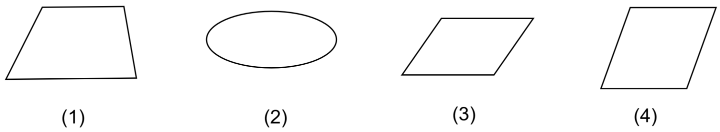
<table>
<tr><th>方法 1 (d_1)</th><th>方法 2 (d_2)</th></tr>
<tr><td>1. 为了识别 直角三角形
(a) 从 直角三角形的定义中分离出来最关键的特征——有一个 90° 的角。
(b) 在大脑中在 三角形的基础上来运用这个特征——检查一下是否有一个角是 90° 的。
(c) 按照下面的规则做出来结论：
i. 如果一个 三角形有一个 90° 的角，则这个三角形就是 直角三角形。
ii. 如果一个 三角形没有一个 90° 的角，则这个三角形就不是 直角三角形。</td><td>1. 为了识别一个 对象是否属于某个类别：
(a) 从这个 类别的定义中分离出来最关键的那些特征们。
(b) 在大脑中去检验需要做判断的对象是否具有 所有这些个特征。
(c) 按照下面的规则做出来结论：
i. 如果这个 对象具有所有的特征，就像定义中所规定的，则判断 这个对象属于这个类别。
ii. 如果这个对象没有 至少一项这些特征，则判断 这个对象不属于这个类别。</td></tr>
</table>

我们建议读者把这里的方法 2——用一步步的方式——用来识别下面的几何图形。在这里，我们的任务是识别出来菱形。菱形的定义是“一个四边都相等的平行四边形”。

下面有几个几何图形，我们需要来确定一下其中哪些是菱形（如果有的话）：

现在，回到之前的直角三角形的识别的任务，去识别一下图 1 中哪些是直角三角形。读者就会发现，这里的方法 2，可以用于识别直角三角形和菱形。

这说明方法 2 比只能用于识别直角三角形的方法 1 要具有更大的通用性。



为了帮助读者更好地欣赏到方法 2 的一般性，我们建议读者再来试试把它用于识别语法单位——子句。

子句的定义是：“一群单词，其中由主语和谓语”。我们需要从下列句子中选出来子句：

(a)“上帝啊!”;

(b)“请原谅我”;

(c)“当 Peter 走进这个屋子的时候”;

(d)“我非常喜欢这本书”。

6.6　方法 2 有多么一般？

述评：提出进一步问题：方法 2 够一般吗？作者看来方法 2 仅仅考虑了纯“和”这个逻辑关系的条件。

浏览一个数学课程，我们会遇到以下的规则：“一个数可以被 5 整除如果这个数的末位数是 5 或者 0”。为了检验方法 2 的一般性，让我们来试试判断几个数(15、17、20 和 23)——按照方法 2——是不是可以被 5 整除。我们建议你把这个练习做一下。如果你做了，就会得到错误的结论。[2]

[2]译者注，后面会发现，这个错误其实是方法 2 之中的“满足所有特征才算”和“不满足其中之一的特征就不算”导致的——在这里命题之中包含了“或”的关系而如果坚持用“满足所有特征才算”就出了问题。反过来，如果我们把特征本身就提炼为包含“或”关系，则坚持用方法 2 就没有任何问题。实际上，我们还会面对由特征之间更加复杂的逻辑关系定义的概念，表述的命题。因

为什么会这样？显然，是因为方法 2 不够一般。

我们如何才能找到更一般的方法呢？我们已经注意到方法 2 对于某些概念和命题是适用的，但是不适用于其他一些概念和命题。

6.7 为什么方法 2 不是总管用？

述评：回答问题：需要考虑了纯“或”这个逻辑关系的条件，得到方法 3。

为了搞清楚这个问题，我们来比较一下方法 2 管用的和不管用的概念和命题。它们有什么不一样呢？很明显，菱形和子句的特征是用逻辑连词“和”（同时满足）相连的，而被 5 整除的特征是由逻辑连词“或”（只要满足一个）相连的。很明显，方法 2 只能用于“和”关系的概念和命题，而不能用于“或”关系的概念和命题。于是，我们就有了方法 3——一个专门用于具有“或”关系的特征的概念和命题的方法。

这是方法 3 和方法 2 的比较：

表 2 方法 2 和方法 3 的比较

此，按照每一种逻辑关系来“构建”一种方法是不可取的。让特征整体就包含了逻辑关系在里面才是更加可取的。

<table>
<tr><th>方法 2（“和” 关系）d_2</th><th>方法 3（或关系）d_2</th></tr>
<tr><td>为了识别一个 对象时都属于或者不属于某个类别:

1. 从这个 类别的定义中分离出来最关键的那些特征们。

2. 在大脑中去检验需要做判断的对象是否具有 所有这些个特征。

3. 按照下面的规则做出来结论:

(a) 如果这个 对象具有 所有的特征，就像定义中所规定的，则判断 这个对象属于这个类别。

(b) 如果这个对象没有 至少一项这些特征，则判断 这个对象不属于这个类别。</td><td>为了识别一个 对象时都属于或者不属于某个类别:

1. 从这个 类别的定义中分离出来最关键的那些特征们。

2. 在大脑中去检验需要做判断的对象是否具有 至少一个这些特征。

3. 按照下面的规则做出来结论:

(a) 如果这个 对象具有 至少一个这些特征，就像定义中所规定的，则判断 这个对象属于这个类别。

(b) 如果这个对象没有 任何一项这些特征，则判断 这个对象不属于这个类别。</td></tr>
</table>

读者很容易就可以检验得到方法 3 适用于“或”关系的概念和命题。这是教科书里面的一些例子：“大小或者形状的概念是一种物理变化”；“不直接对象回答了“给谁”或者“给什么”的问题”；“描述一个东西看起来、感觉起来、尝起来、听起来怎么样的词是形容词”。

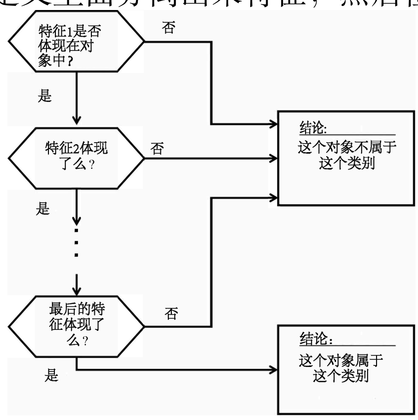
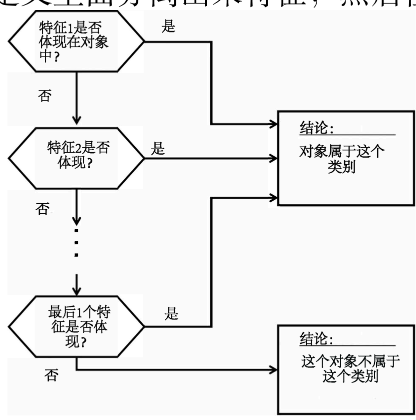
那么，方法 3 比方法 2 更加一般吗？不是的。如果方法 3 包含了方法 2，那它就更加一般了。但是，这里不是包含关系，而是补充关系。这表示它们具有相同的一般性程度。

6.8　方法 2 和方法 3 的图示

述评：方法 2 和方法 3 的对比。

流程性方法的一个高效的表示方式是画流程图，见表 3。[3]

表 3

<table>
<tr><td>方法 2
应用于“和”关系的概念
识别对象是否属于某个类别，其特征用逻辑连词“和”相连</td><td>方法 3
应用于“和”关系的概念
识别对象是否属于某个类别，其特征用逻辑连词“和”相连</td></tr>
<tr><td>为了判断某个对象是否属于某一类，从定义里面分离出来特征，然后检查

</td><td>为了判断某个对象是否属于某一类，从定义里面分离出来特征，然后检查

</td></tr>
</table>

6.9　方法 2 和方法 3 合起来足够一般性可以处理条件中的任何逻辑结构了吗？

述评：进一步提出问题：方法 2 和方法 3 合起来也不够。

[3]组织和呈现心智过程以及其他流程的流程图最近被叫做“图形组织者”（见，例如 S. Parks & Black, 1990, 1992）。

当我们检验方法 2 和方法 3 的一般性的时候，我们遇到了下面两类命题：

1. 社会保障管理局的一项规定："只有未婚母亲，她们有一个受抚养的孩子，而且收入不超过 $ 12000，或者已婚母亲，她们有两个受抚养的孩子，家庭收入不超过 $ 16,500，才有资格获得这项福利。"

2. 语法定义："动词是通常作为谓语的语法中心的一个词，它表达一个动作、事件或存在的方式"（韦伯斯特商务通信指南，1988 年，第 217 页）。

根据这个规则，我们如何来决定一个女人是否应该获得这样福利？你是怎么来做这个决定的？你是怎么来开展逻辑论证的？类似地，你怎么判断一个词是不是动词？你怎么来开展逻辑论证的？

方法 2 和方法 3 都不能用于这样的命题的判断，因为这些命题既包含了"和"关系也包含了"或"关系。换句话说，"享受这项福利待遇"的条件的逻辑结构和"动词"的条件的逻辑结构都不是纯粹的"和"关系，或者纯粹的"或"关系，而是具有混合关系结构。然后，方法 2 和方法 3 仅仅是为了纯粹的"和"关系，或者纯粹的"或"关系而设计的，它们处理不了混合关系。因此，它们还不够一般，我们需要更一般的方法。

6.10 一种识别具有嵌套的混合逻辑结构的复杂概念和命题的方法

述评：运用逻辑符号来构建表达式，得到方法 4（把"和关系"的条件做了"或关系"）和方法 5（把"或关系"的条件做了"和关系"）。

我们用前面提到的社会福利的例子来展示这个更加一般的方法

1. **操作 1:** 把原本分类形式的命题表述为 if...then（如果... 那么）的形式

   "如果一位未婚母亲有一个受抚养的孩子且她的收入不超过 $ 12000，或者一位已婚母亲有两个受抚养的孩子且她的家庭收入不超过 $ 16,500，那么，且仅当那时，她才有资格获得所讨论的福利。"

2. **操作 2:** 为揭示转换后命题的内部逻辑结构，请使用括号来描述它。

   "如果（一位未婚母亲有一个受抚养的孩子且她的收入不超过 $ 12000）或者（一位已婚母亲有两个受抚养的孩子且她的家庭收入不超过 $ 16,500），那么，

且仅当那时，她才有资格获得所讨论的福利。”

3. **操作 3:** 将揭示的逻辑结构以图形“逻辑图”形式呈现，这使得结构更加透明和鲜明。

逻辑图

如果

I. a. 一位未婚母亲有一个受抚养的孩子 **和**

b. 她的收入不超过 $ 12000

或

II. a. 一位已婚母亲有两个受抚养的孩子 **和**

b. 她的家庭收入不超过 $ 16,500

那么, 且仅当那时，她才有资格获得所讨论的福利。

1. **操作 4:** 用命题逻辑的公式表达这个命题（无论是以句子形式还是逻辑图形式），它以最概括的形式简洁地描述了其逻辑结构。

让我们用字母 a 表示条件 I(a)，用 b 表示条件 I(b)，用 c 表示条件 II(a)，用 d 表示条件 II(b)，并且用 E 表示结论“她有资格获得所讨论的福利”。我们进一步用符号 & 表示逻辑连接词“和”（and），用 $\vee$ 表示逻辑连接词“或”（or），用 $\rightarrow$ 表示单向的“如果... 那么（if..., then）”连接，用 $\leftrightarrow$ 表示双向的“如果... 那么（if..., then）”连接。

于是，按照命题逻辑的语言，我们的命题将被表示为：

$$(a\&b) \vee (c\&d) \leftrightarrow E.$$

这个命题读作：如果 a 和 b 同时满足，或者，c 和 d 同时满足，则我们有结论 E。

我们建议读者将刚刚描述的方法应用到动词的定义上，然后比较两个命题的逻辑结构。完成之后，将明显看到，虽然第一个命题是把“和关系”的条件做了“或关系”，第二个命题是把“或关系”的条件做了“和关系”。

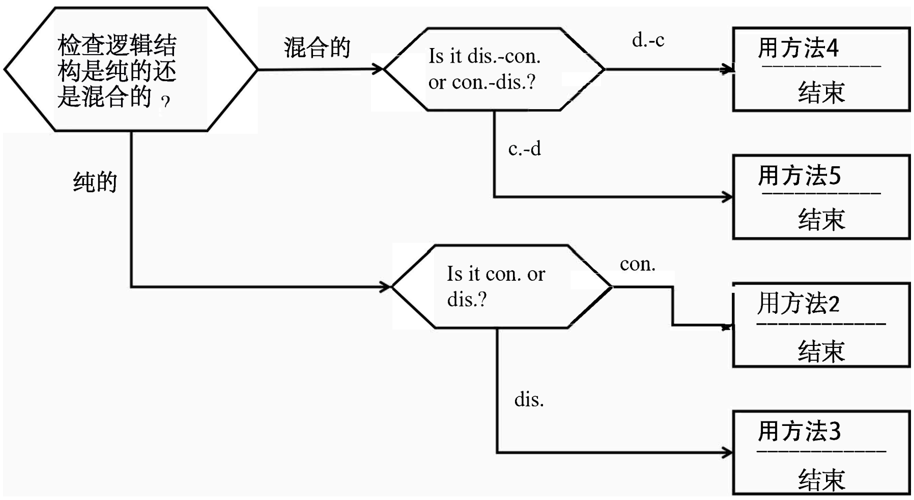
现在我们建议读者自行找出一个把“和关系”的条件做了“或关系”的概念和命题的判断方法（方法 4）以及一个把“或关系”的条件做了“和关系”的概念和命题的判断方法（方法 5）。这些方法（d_3）将比方法 2 和方法 3（d_2）拥有更高程度的普遍性。

6.11　还是需要一个更加具有一般性的方法

述评：按照是否纯逻辑还是混合逻辑，以及混合的方式，综合运用方法 2,3,4,5，得到方法 6。

现在我们来构建一个更加具有一般性的方法（d_4），来把之前得到的所有方法都整合起来。

以下是它可能的样子：



6.12　方法 6 是最一般的方法了吗？我们来看一看

述评：进一步提出问题：方法 6 足够一般了吗？可以用来指导运用一般的命题了吗？作者认为还是不行，由于单向命题和双向命题的区别。但是，吴金闪认为，实际上这个区别没必要有。

这些已经前面建构起来的方法是用来识别概念的，这些概念的特征具有不同的逻辑结构。但是，概念仅仅是命题的一种形式。其他的命题还有规则、公理、定理、自然规律（物理的、化学的、生物的，等等），有关对象属性及其关系的陈述，

以及一些其他命题。学生在学校的学习中会遇到所有这些命题。他们应该学会并知道如何应用它们。[4]

为了方法 6 成为应用知识的最通用方法，它必须适用于所有知识，包括规则、规律和其他类型的命题。这样的方法必须适用于所有这些命题。那么，方法 6 和它所依赖的那些较不通用的方法是否适用于所有类型的命题呢？让我们来测试一下。

让我们考虑一个关于正方形的一个属性的简单几何命题（定理）："正方形的对角线是垂直的"。

以条件如果... 那么（if...then）形式表述："如果一个几何图形是正方形（S），那么它的对角线是垂直的（dp）"。

用命题逻辑的语言表达：$S \to dp$。

这显然是一条正确的陈述。现在让我们来逆向它："如果一个几何图形的对角线是垂直的，那么这个图形是正方形：$dp \to S$。这个陈述不是真的，因为有垂直对角线的图形也可能是菱形，而不仅仅是正方形。

因此，我们例子中的陈述 $S \to dp$ 是真的，但逆向陈述 $dp \to S$ 则不是真的。

只有一种命题，无论哪个方向都永远是真的，那就是定义。其他命题可能在一个方向上是真的，但在另一个方向上可能真也可能不真。它们在另一个方向上的真假必须根据每个具体命题来确定。

迄今为止所描述的方法在对定义和那些在两个方向上都是真的双向命题方面只是一般性的。但这些方法不适用——或者不完全适用——于只在一个方向上是真的单向命题。这意味着所描述的方法仍然不是最通用的。

应用单向命题的方法与应用双向命题的方法在部分上是不同的。在下面的表格中，我们将描述纯"和"关系（方法 2a）和纯"或"关系（方法 3a）结构在单向命题中的方法。我们建议您将它们与为双向命题设计的相应方法 2 和方法 3 进行比较，以了解差别。显然，既然我们需要方法 2a 和方法 3a，那就意味着我们也需要方法 4a 和方法 5a，读者可以通过修改方法 4 和方法 5 来轻松创建这些方法。

[4]译者注，这里我们的任务已经从识别一个对象是否属于某个类别，也就是概念识别，变成了判断一个命题是否成立。实际上，更简单的方式是，先学会判断一个命题是否成立的方法，然后，对于定义不过就是当作双向都成立的命题去判断两次——每个方向判断一次。其他的双向都成立的命题也是如此。用后面的符号，也就是说我们只需要方法 6a：对于单向成立的命题，就运用 6a；对于定义以及双向成立的命题，对两个方向的命题，分别用 6a。再注意到，方法 3、4 和 5 也是没有意义的（只需要把条件部分当作一个包含了逻辑运算的整体，而不是一项项条件分开讨论再做逻辑运算），只需要方法 2，或者说，我们需要的是单向命题是否成立的判断方法 2a。

6.13 判断单向命题是否成立的方法——仅仅包含纯“和”关系（方法 2a）和纯“或”关系（方法 3a）

述评：针对单向命题纯逻辑关系，提出方法 2a 和 3a。同样，吴金闪认为这完全没必要。

表 4

方法 2a （纯“和”关系单向命题）d_2	方法 3a （纯“或”关系单向命题）d_2
为了识别一个对象是否属于某个类别或者决定是否进行由“如果... 那么”的后半句所指的某个操作： 1. 从命题的“如果... 那么”的前半句里面分离出来特征和条件。 2. 把这个特征或者条件放到这个对象或者情景之中去检查这个对象或者情境是否具有这些特征。 3. 根据以下规则来得到结论：（a）如果这个对象或者情境具有前半句中的所有的这些特征满足前半句中的所有的条件，则这个对象属于后半句中指的这个类，或者进行后半句指定的操作。（b）如果其中一个特征或者条件没有满足，则我们 **不能得到右侧的结论，不能判断其是否属于右侧指的这个类别，不能判断是否需要进行右侧制定的操作**。	为了识别一个对象是否属于某个类别或者决定是否进行由“如果... 那么”的后半句所指的某个操作： 1. 从命题的“如果... 那么”的前半句里面分离出来特征和条件。 2. 把这个特征或者条件放到这个对象或者情景之中去检查这个对象或者情境是否具有这些特征。 3. 根据以下规则来得到结论：（a）如果这个对象或者情境具有前半句中的所有这些特征之中的至少一项满足前半句中的所有的条件之中的至少一项，则这个对象属于后半句中指的这个类，或者进行后半句指定的操作。（b）如果其中所有的特征或者条件没有满足，则我们 **不能得到右侧的结论，不能判断其是否属于右侧指的这个类别，不能判断是否需要进行右侧制定的操作**

方法 2a 中使用 3b 的一个例子. 假设有人规定了以下带有在其右部分指明的行动的“如果... 那么（if..., then）”规则：“如果正在下雨，离家时带上雨伞”。现在假设天气晴朗并没有下雨。那么我是不是就不必执行行动，即不需要带雨伞呢？

不一定。如果我预期一天中晚些时候会下雨，我仍然可能会带上雨伞。这条规则说明了如果条件出现应该做什么，但没有说明如果条件不存在应该怎么办。规则没有说明如果没有下雨该做什么。于是，决定是开放性的，既可以带上也可以不带上雨伞。

方法 3a 中使用 3b 的一个例子. 假设有人制定了以下带有在其右部分指明的“if..., then”规则：“如果正在下雨或你预计一天中晚些时候会下雨，离家时带上雨伞”。现在假设天气晴朗并没有下雨，也没有预报说会下雨。那么我是否就一定不需要采取行动，即不带雨伞呢？不一定。我可能因为其他某些原因仍然会带上雨伞。这条规则说明了如果至少有一个条件出现时应该做什么，但它并没有说明如果两个条件都不存在时应该怎么办。于是，决定是开放性的，既可以带上也可以不带上雨伞。

很明显，对方法 2 和方法 3 进行的将它们转变为方法 2a 和方法 3a（以适应单向命题）的修改，也应该在所有其他方法（4 到 6）中进行，成为方法（4a 到 6a）——基于方法 2a 和方法 3a。我们建议读者们自行进行这些修改，并且明确表述出来方法 4a 到方法 6a。

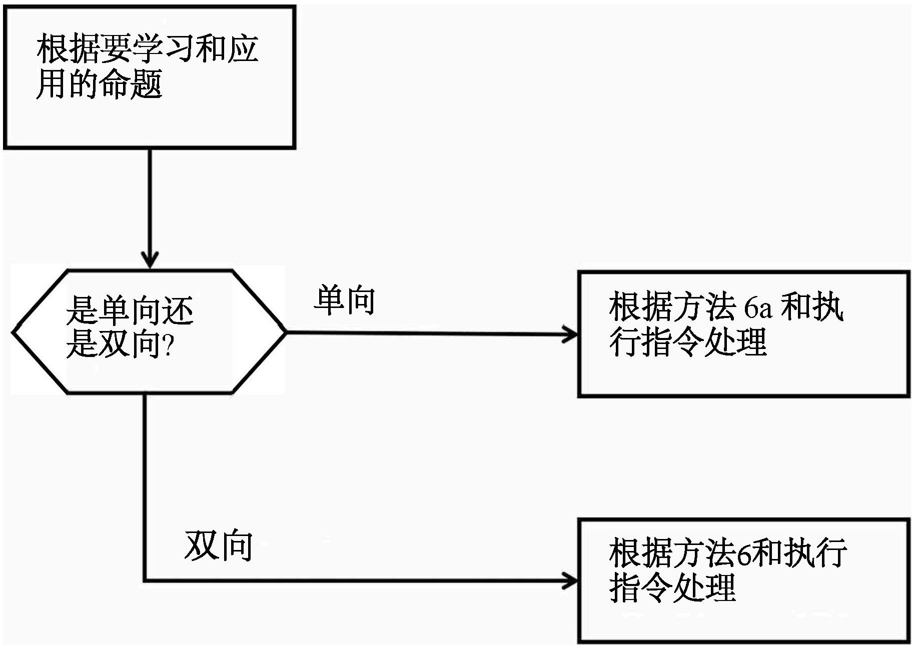
6.14　方法 7（最一般的）(d_5)

述评：从 2a 和 3a 到方法 6a，综合运用方法 6 和 6a，得到最具有一般性的方法 7。同样，吴金闪认为这完全没必要。

现在，我们终于得到了用于学习和应用任何以任何类型的命题（定义、规则、定理、法律等）表达的概念性知识的最通用的方法。这些命题在如果... 那么（if...,then）陈述的左部分可能具有任何特征或条件的逻辑结构，并且在左右两部分之间可能有两种连接方式（双向或单向）之一。

以下是应在方法 6 和方法 6a 之前执行的指令，合起来使得方法 7 成为最通用的最具有一般性的方法：

注意：在本章描述的方法所涵盖的命题中，前项条件和后项结论之间的联系是决定性的。也有一些命题是具有概率性联系的，在这些命题中，后项以一定的概率，而不是肯定地，从前项中得出。这就是为什么方法 7 只有在确定性命题方面



才是最具有一般性的。[5]

6.14.1　教授学生应用概念和命题知识的最通用方法有多困难？

述评： 从 2a 和 3a 到方法 6a，综合运用方法 6 和 6a，得到最具有一般性的方法 7。同样，吴金闪认为这完全没必要。

从对最通用方法 7 的描述，以及我们建构出来它的方式中，可能会产生一种

[5]译者注，按照先把条件看作一个包含了子条件和子条件的逻辑运算，再把命题永远只从单向来考虑（需要双向就做两次单向）的方式，最具有一般性的方法是方法 2a。这样逻辑上更简单和顺畅。不过，这不影响本文的主旨——对于判断命题是否成立这么一件看起来很简单的事情，我们需要对这个过程中所需要的心智操作做详细的分解，然后把分解以后的所有操作整合起来，做到内化和自动化，只有这样经过分解-内化-自动化的过程，才能帮助学生真的学会思考；学生不能直接提升智慧，只有通过这样的可以分解-内化-自动化操作，掌握了这些操作，有了方法，才能提升。更进一步，我们说，对于更加复杂的心智活动，那更加需要通过分解-内化-自动化来形成可以学会的方法了。同时，这文章也隐含了这样的意思：判断命题是否成立以及更加复杂的心智活动，是可以被分解-内化-自动化的，可以被学生掌握的，尽管针对任何具体任务其心智活动的这个分解-内化-自动化的过程可能并不简单。

印象，即教授和学习这种方法是一个困难和漫长的过程。实际上，这是一个容易且相对快速的过程，学生们非常享受这个过程。此外，甚至有可能教授初中学生如何独立发现概念和命题的基本逻辑结构以及处理它们的方法。在（Landa, 1974）的第 15 章中，详细描述了这样的一节课。我们强烈建议读者熟悉那节课，因为它几乎提供了如何进行教授的脚本。开展这样的课程的方法论在那里是被精确和结构化地做了描述，它几乎代表了一个教学算法，任何老师都能照着用起来。

在原始的俄文 1966 年版的书籍（Landa, 1974）出版后，我们对小学生也进行了一些初步的实验。结果显示，较小的孩子们也可以被教授概念和命题的基本逻辑结构以及处理它们的方法。

教授学生一些在下一节中讨论的辅助方法需要更多的时间。

6.14.2　命题的显性和隐性逻辑结构。为什么需要教授和学习辅助方法以使一般性方法能够发挥作用

述评：为了让这些方法发挥作用做了补充说明，关于从自然语言到逻辑语言的转换。

在我们用来构建应用知识的最通用方法的大多数例子中，逻辑连接词“和”(and)、“或”(or)、“非”(not) 和“如果... 那么”(if..., then) 都出现在了命题中，因此它们是显性的。这使得辨识逻辑结构及其在逻辑图表形式中的表达相对容易。然而，在科学和日常语言的许多命题中，逻辑连接词是由语法连接词表达的，或者根本不存在，这使得它们及其相关的逻辑结构是隐藏的或隐性的。

兰达法发展了一系列把隐藏的或者隐性的逻辑结构显性化的方法。由于空间所限，我们不在这里呈现了。在这里，我们仅仅提供几个例子来看一些这些方法大概是什么。

把语法中的逻辑关系连词转化为逻辑学中的逻辑关系

语法中的逻辑关系连词	转化为逻辑连词
1. 他聪明但是懒。	1. 他聪明并且懒。
2. 除非他道歉，我不会原谅他。	2. 如果他不道歉，我不会原谅他。 也意味着 [7]：如果他道歉了，我会原谅他。
3. 抢银行的告诉银行柜员，“保持安静，你会没事”。	3. 如果你保持安静的话，那么你将会没事。

隐性的逻辑结构显性化的例子

隐性的逻辑结构	显性化了的逻辑结构
1. “分词短语是含有分词的一组相关词汇。”（Warriner & Griffith, 1957, p. 37）	1. 当且仅当一组词汇（a）和一个分词相关 **并且**（b）包含一个分词的时候，这组词汇被称为分词短语。

7 发现、教和学一般性的思考方法的教育价值

述评：教这些一般性的思考的方法的好处。

教和学通一般性的思考方法有以下重要的教育价值：

- 它为学生提供了统一和普遍的工具，以获取、操作和应用所有学科的任何内容的知识。
- 它只需要教和学每个方法一次，从而无需针对每个特性的知识来教和学其获取、操作和应用。
- 它节省了大量的时间，从而极大地提高了教和学的效率。
- 它极大地提高了所获得知识、技能和能力的质量。
- 它显著减少了教和学的困难。
- 它预防了许多错误或极大地减少了错误率。

- 它几乎能把每个人培养成具有专家水平的学习者和问题解决者，并且这样做是可靠和相对快捷的。

7.1　教和学一般性的思考方法的额外的教育益处

述评：教这些一般性的思考的方法的好处。

以下是一些额外的但极其重要的，因一般性的思考方法而衍生的教育益处：

- 学生开始理解知识的一般构成和结构——任何知识——不论它特定的领域和内容如何，这导致跨学科思维的发展。
- 学生获得了一个用于结构性分析和比较知识的强大工具，不论其特定的内容和领域如何。
- 他们获得了一个工具和能力，能够从特定（具体）中看到共同（普遍）。
- 他们开始更容易将知识、心智操作及其系统（无论通用的或者更具体的方法）从一种内容转移到另一种内容，无论是在同一学科内还是在不同学科领域间；转移的范围变得无与伦比地宽广。
- 他们开始意识到自己的思维过程，并获得了工具及能力来自我管理、自我调节和自我控制这些过程。他们的思维变得真正自足和独立。
- 他们发展出一般性的处理不同问题的方法，无论在同一学科或不同学科知识领域内。

7.2　为什么一般性的思考的方法在今天的学生不经常被教？

述评：为什么学校不这样教，不教这些。

原因有以下几个：

1. 教育科学的成熟度不足，它还未意识到对所有年龄段的学生教授一般性的思考的方法的关键重要性。
2. 在教育学和心理学中一般性的思考的方法的发展不足，导致缺乏对不同的一般性的思考的方法的构成和结构的科学认识。
3. 教育学和心理学中对一般性的思考的方法的教学方法的研究发展不足。
4. 在教学实践中，注重于教授和学习具体的知识和技能，而非普遍知识获取、操作和应用的一般方法；按照兰达法的观点，具体的知识和技能应该基于这些通用方法来进行教和学。
5. 大多数教师缺乏——但是各个领域中的许多专业人士和专家级问题解决者往往拥有——对自己的心智过程和思考的方法的认知，这使得这些方法的沟通交流以及向学生传递这些方法几乎成为不可能。
6. 教师培养和训练的缺陷导致师范生和在职教师既不学习一般性的思考的方法（以及其他认知活动的方法），也不学习教授一般性的思考的方法的方法。

7.3 由于不教授学生一般性的思考方法而导致的学习和思考问题的简要总结

述评：不这样教，不教这些的坏处。

以下是由于不教授学生一般性的思考方法，也不教给学生自己去发现这样的方法而导致的学习和思考问题的简要总结：

1. 如果不教授一般性的思考方法，学生将被迫尝试自己去发现它们。
2. 如果不教授发现一般性的思考方法，学生只能采用他们唯一可用的方法——试错。
3. 通过试错来发现一般性的思考方法是一个困难的过程（因而导致学习和思维中的困难和问题）。

4. 通过试错来发现一般性的思考方法是一个漫长的过程（因此每个特定主题的教学和学习时间过长）。

5. 通常情况下，通过试错来发现一般性的思考方法是一个不系统和没有计划性的过程。

6. 所发现的方法很多时候是基于经验拓展做一般化归纳得来的，往往不够一般化（它们只能实现有限的可转移性和有限的应用范围）。

7. 很多时候，并非所有组成行动都可以被发现，结果是发现的方法在一个或多个方面存在缺陷（不完整、低效率等）。

8. 就算所发现的方法正确且足够普遍的，它们往往效率低下（不经济）。

9. 通过试错发现方法（M_a）的学生通常不能明确地意识到和表达出来这些操作，而不能达到显性的表达出来的方法的水平（M_p）。结果是学生无法自我管理、自我调节和自我控制他们的心理过程。

10. 由于对心智操作（M_p）的无意识，学生无法向其他人传达他们的心智过程系统（M_a）。

8　一般性的思考的方法是内容无关的吗？

述评：对一般性的思考的方法的一般性的再一次补充说明：适用于所于具体知识，但是需要区分逻辑结构的类型。同样，吴金闪认为，这个区分完全没有必要。

如果以内容指的是使得三角形与菱形或名词之间有所区别的特征，那么答案是肯定的。但是，如果在内容的概念中也包含这些特征的逻辑结构，则答案是否定的。内容的逻辑结构也是一种内容，虽然其性质截然不同。方法不受第一种类型的内容所决定，但却被第二种类型的内容所决定并反映了这一内容。[6]

[6]译者注，按照我们一定提到的，先把条件看作一个包含了子条件和子条件的逻辑运算，再把命

这样的一般性的思考的方法的力量在于，它们允许人们识别出来第二种类型的内容，并在心智上将其与第一种类型的内容分离。这使得可以将心智操作应用于任何第一种类型的内容，即使是过去从未遇到的内容。从而，这样的一般性的思考的方法的帮助下，人们能够超越自己过去经验的限制，有效地思考那些他们没有先前个人经验的事物。

9 一般性思考的方法与智力

述评：讨论了智力和一般性思考的方法的关系。

终于，美国和一些其他国家的认知心理学得出了智力是可以教授和学习的论点（例如，可参考 Wimbey Wimbey, 1975; Sternberg, 1983; Perkins, 1995）。（顺便说一句，这一论点在苏联的认知与教育心理学中几十年前就提出了）。然而，具体哪些，什么样的过程或机制是可以教授的？在没有对这个问题给出清晰准确答案之前，智力的教授性和可学习性的论断仍悬而未决。为了了解如何教授——或者说引发、产生——智力，有必要明确智力到底是什么。

根据兰达法，一般智力无非就是对一套最具有一般性的思考的方法的掌握（不仅是知识或者不只是知识！），这些方法适用于任何特定内容。

那么，教和学智力意味着什么呢？

它意味着，根据兰达法，教和学一般性的思考的方法，它们会促进一般性的智力的形成发展。这里多说一句：智力不是哪些方法（M_a）对应的操作或者操作的执行，而是执行了这些操作之后在大脑里面剩下的东西。或者说，智力是之前的操作留下的“痕迹”、后效。

或者，我们这样来表达：智力是不能教和学的，只有方法可以；智力只有通过掌握和执行方法以及相应的操作来形成。[7]

题永远只从单向来考虑（需要双向就做两次单向）的方式，最具有一般性的方法是方法 2a。因此至少在判断命题是否成立这一点上，思考的方法是内容无关的。对于其他任务，是否也可以梳理出来一般性的思考的方法，实现内容无关，那是另一个话题。

[7]译者注。只有可操作的才是可以教和学的。大多数人同意程序性知识——其实就是一系列操作——是可以教和学的，把事实性知识先记忆下来然后未来需要的时候按照某个线索召回则事实性

10 局限

述评：对本论文还没有考虑到的因素（概率性命题判断）的讨论。

在本章中，我们只讨论了基于对知识应用对象的完整信息的决定性的知识应用方法。然而，还有一些概率性的认知活动和思维的通用方法，它们是概率性直觉判断的基础。关于认知活动的概率方法及其教学方法的讨论则是另一个话题了。

11 参考文献

述评：列出本文参考文献。

Hirsch, Jr., E. D. (Ed.).(1993). What your 6th grader needs to know. Fundamentals of a good sixth-grade education (p.61). New York: Delta.

Lando., L.N. (1974). Algorithmization in leartning and instruction. Englewood

知识也就成了操作，于是也是可以教和学的。也正是如此，才导致当前的教和学中，所学习的大部分内容都是事实性程序性知识——容易学、容易考。但是，我们都知道，真正的提出和解决问题靠的不是事实性程序性知识，而是靠的对学科的理解，也就是做到“像一个个学科的专家那样的提出和解决问题，来进行思考”。于是，教和学学科理解学科思维层面的知识是绝对有必要的。这个必要性没有人怀疑。问题在于，这样的知识能教和学吗？如果把这样的知识当作事实性知识来学，固然学生可能可以背诵可以完成填空题，但是，学生真的能够掌握学科理解和学科思维吗？例如，“数学是思维的语言，是描述世界的语言，数学研究结构以及结构之间的关系，数学主要运用抽象和演绎证明（包含计算）来提出和解决问题，数学命题是数学结构之间的关系其本身需要证明其运用需要判断前提是否满足”等等这些话直接教给学生是没有用的。按照兰达法，我们一旦想办法把这些关于学科理解和学科思维的知识变成一个流程，一个解决问题的流程，那么，学生一旦学会这个流程，内化和自动化之后，就有可能可以掌握学科理解和学科思维层面的知识。除了针对问题和任务来分解-内化-自动化思考的方法，这篇文章的另一个值得参考的地方，就是对问题不断地进行一般化。因此，合起来，这篇文章留给我最大的启发就是：把学科理解和学科思维层面的知识转化为解决问题的流程，并且不断地把这个问题一般化使得其覆盖的问题场景更多，从而使得学生掌握这个流程之后可以迁移到同类型的问题上去，从而帮助学生看起来发展了智力。

Cliffs, NJ: Educational Technology Publications.

Landa, L. N. (1983). The algo-heuristic theory of instruction. In Ch. M. Reigeluth (Ed.), Instructional-design theories and models: An overview of their current status. Hillsdale, NJ: Lawrence Erlbaum Associates.

Landa, L. N. (1997). The algo-heuristic theory and methodology of learning, performance, and instruction as a paradigm. In Ch. R. Dills A.J. Romiszowski (Eds.), Instructional development paradigms. Englewood Cliffs, NJ: Educational Technology Publications.

Parks, S., Black, H. (1990, 1992). Organizing thinking. Graphic organizers. Books 1 and 2. Pacific Grove, Calif.: Critical thinking press software.

Perkins, D. (1995). Outsmarting IQ: The emerging science of learnable intelligence. New York: Free Press.

Sternberg, R. (1983). How can we teach intelligence. Research For Better Schools

Warriner, J. E. Griffith, F. (1957). English grammar and composition. New York: Harcourt, Brace World, Inc.

Webster's guide to business correspondence (1998). Mertriam-Webster Springfield, Mass.: Inc., Publishers.

Whimbey, A. Whimbey, L.S. (1975). Intelligence can be taught. York: E.P. Dutton.

Zykova, V. I. (1963). The formation of practical skills at geometry lessons (in Russian). Moscow: APS publishing house.

第二部分

Landamatics Instructional Design Theory and Methodology

1 ABSTRACT

This paper demonstrates how to design an instructional process aimed at teaching general methods of thinking, using the Landamatics theory and methodology. Landamatics is not a collection of lesson plans, but rather a general method of approaching the design of any effective course of instruction or any lesson plan. The method formulates general procedures to apply to teaching any specific knowledge and any cognitive process. Analysis of the problems many students have in being able to solve problems similar to those they were taught is that they have not been taught a general method of reasoning as a system of general instructions. In contrast to the empirical generalizations formed in the minds of student who have had conventional instruction, the Landamatics approach forms reliable, scientific, concept-congruous generalizations. This is illustrated through the example of teaching students to recognize right triangles. Several strategies are available to the teacher. One is to lead the students to make independent discoveries of the concept and the method of applying it. Another is to give the students all the information possible about the concept, and a third is to combine these two approaches. The teacher's strategy is chosen according to the objectives desired, but the first strategy

appears to be the most valuable. Central to the Landamatics method are getting students to discover and realize the system of mental operations involved in the application of the concept and its definition, and then getting them to formulate a corresponding system of instructions. Providing practice and opportunities for the internalization of the method also follow in the Landamatics approach. This will bring about automatization of the mental operations of the method. Generalizations of this approach through several forms result in a method that can be applied to concepts with different logical structures of their characteristic features. Advantages of this general method are discussed. (Contains 4 tables, 3 figures, and 11 references.)

2 Preface

The approach to designing instruction for teaching and learning general methods of thinking, described in this chapter, was developed within the framework of the Algorithmico-Heuristic Theory (AHT) of performance, learning, and instruction. Since systems of algorithmic and heuristic instructions, and their corresponding systems of mental operations, represent general methods of thinking, the AHT was, in essence, a theory and methodology of teaching and learning general methods of thinking.

The first studies of the author and his associates were focused on thinking processes and methods involved in knowledge application, i.e., were studies of methods of applying knowledge. Three things, however, soon became clear. First, that methods of thinking are a particular case of a more general methods of cognitive activity which include methods of perception, methods of memorization and some others. Second, that methods of thinking are not limited to the methods of applying knowledge but include methods of acquiring knowledge as well or, stated more generally, methods of learning. Third, that learning methods of knowledge application enormously affects the process of knowledge acquisition making it vastly more effective. The reason for this is simple: people don't learn knowledge just by listening to or reading the explanations (unless, of course, their aim is just mere familiarization with some knowledge). They learn it by applying it to solving problems, which is one of the main purposes of the process of practicing. If this is so, then it becomes apparent why effective methods of knowledge application make the process of knowledge acquisition much more effective: with properly designed instruction, the knowledge application process becomes an important component of the process of knowledge acquisition.

After the first studies had been conducted, it also became clear that teaching methods of cognitive activity affects not only the processes of learning and thinking but leads to the formation of certain qualities of mind and personality traits - such, for example, as intelligence, intuition, self-management, self-regulation and self-control, good organization of mind, a higher level of confidence in the ability to learn and solve problems, and such personality traits as systematicity in approaching problems, thinking of possible strategies for attacking problems and analyzing them before trying to actually solve them, and some others.

In short, the subject of the AHT turned out to be much broader than the term ”algo-heuristic” theory and methodology suggested. The AHT became, in fact, a rather general and comprehensive theory of performance, learning, and instruction. Seeing the discrepancy between the actual broader subject of the theory and the narrower subject that the term ”algo-heuristic” suggested, Prof. Berkowitz of CUNY coined the name ”Landamatics” which we, initially, were hesitant to accept and use. Because, however, this name is beginning to be used on an increasingly wider scale (even on the Internet), we will employ it here as well.

Landamatics is not a theory of learning and instruction that indicates how to teach one or another specific topic, concept, or skill. It is not a collection of effective lesson plans. Rather, it is a general method (or methodology) of approaching the design of any effective course of instruction or any lesson plan, whether the task is to teach knowledge of certain phenomena or a process of visual analysis of an object or a strategy of thinking or anything else. The method formulates general but, at the same time, sufficiently detailed procedures algorithmic or nonalgorithmic which can be applied to designing and teaching any specific knowledge and any cognitive process.

In this chapter, we will demonstrate how to design an instructional process aimed at teaching general methods of thinking, using the Landamatics theory and methodology.

Among different kinds of methods of cognitive activity, for the purpose of illustration we chose, for this chapter, methods of thinking. Among different methods of thinking (such as methods of explanation, methods of proof, methods of drawing inferences from certain premises, etc.) we chose the method of identification of objects as belonging to or not belonging to a certain class on the basis of concepts of those classes and their respective definitions. Later in the chapter, the method will be extended to the method for drawing conclusions not only about objects' belonging to or not belonging to a certain class but also about their attributes and their relationships to other objects. Such conclusions require the application of a broader class of theoretical propositions, such as laws of nature, axioms, theorems, rules, and others.

To summarize, we can say that the intent of this chapter is to formulate and describe, by the chosen example, a general (Landamatics) method of teaching general methods of thinking, i.e., to show not only what to teach but how to teach it as well. To achieve the challenging learning results specified in the chapter, the what to teach and the how to teach is equally important.

A note on the terminology that will be used. Any process of thinking is a process of applying knowledge. The differences between the thought processes are determined, in the first place, by the purpose of knowledge application: whether it is to identify things, or to explain things, or to prove things (statements), etc. The purpose, along with problem conditions, specify how the learned knowledge should be used (applied) for achieving the goal what should be mentally done with it. Although very often one can encounter expressions like "visual thinking", the term "thinking" is used here more in a figurative rather than precise sense of the word. Thinking, in contrast, for example, to perception and imagination, deals with concepts and propositions rather than images, although images are always engaged in the processes of thinking. The notion of applying knowledge is broader than the notion of applying concepts and propositions, as the former also includes the application of images. When, throughout this chapter, we use, for brevity, the phrase "a method of applying knowledge" we will have in mind, mainly, a method of applying concepts and/or propositions.

Let us now consider the following two expressions: "a method of applying knowledge for the purpose of identification" and "a method of identification". These expressions convey the same meaning, for the process of identification is carried out through the application of knowledge for the aim of identifying things. We will use these expressions interchangeably. Also, because in this chapter the process of identification, and its corresponding method of identification, are used only to illustrate by example how to teach any methods of applying knowledge, we will often use a broader term "a methods of knowledge application" rather than a more narrow term "a method of knowledge application for the purpose of identification". This will be done to underscore that what was being said with regard to a method of identification is true for all other methods of thinking, and often, even all methods of cognitive activity.

3 Teaching General Cognitive Processes And Methods Of Thinking As One Of The Most Important Goals Of Education

The fact that recurrently pops up in discussions about the goals and objectives of education is the immense speed of developing new knowledge in a modern, information-based industrial society. Knowledge is changing so rapidly that what we learn today may become outdated and obsolete a decade, or perhaps even a few years, from now.[1]

The following question arises: If knowledge constantly changes in the course of

[1]One has, of course, to have in mind that not all knowledge is changing with time and gets obsolete. For example, the fundamental knowledge like the knowledge about basic laws of mathematics or mechanics, about historical facts and events, about many geographical phenomena and some others are very stable and, in fact, "eternal".

scientific and technological development, do the cognitive mechanisms of acquiring and applying knowledge, in the process of such development, change as well? Or, to be more precise, do they change as rapidly as the knowledge being acquired by mankind? The answer is no.

Experts in any field of scientific, technological or practical activity, who have already learned how to effectively acquire and apply knowledge, use essentially the same cognitive operations and processes (out of some repertoire) to learn and manipulate various knowledges. These processes may be different with regard to different kinds of knowledge (for example, knowledge about facts versus knowledge about laws of nature) and/or with regard to different kinds of problems to be solved, but these proceses are the same with regard to the same kinds of knowledge and problems. Thus, while the knowledge acquired and handled may be variable, the ways methods of its acquisition and handling represent a constant. Because ways (mechanisms) used in acquiring and handling varying knowledge are constant, we can say that, in this sense, these mechanisms are content-independent and therefore general.

If we accept the point that learning how to acquire and apply knowledge - any knowledge, variable knowledge - is as important as learning the fundamental knowledge (and perhaps more important than learning specific knowledges that can soon become obsolete), then teaching students general cognitive processes and their corresponding methods - becomes one of the critical goals of education.

Obviously, teaching general cognitive processes applicable to variable knowledges can be carried out only through and within teaching specific knowledges. At issue is, however, whether teaching knowledge represents an objective in itself or, also, a means of teaching general cognitive processes.

4 An Odd Situation In Education

An odd situation takes place in schools: students are requested to identify objects, explain things, draw conclusions, proof statements, and so on but are not taught -and don't know - what is an identification, what is an explanation, what it means to draw a conclusion, to proof a statement, etc. At issue is not a lack of formal definitions of those processes (a definition would not teach much) but knowledge of the mental operations, and their systems, engaged in those processes. An explanation given by one teacher that "to explain means to make something clear" does not teach anything, as the questions arises what one should mentally do with something to be explained to make it clear.

Numerous interviews we conducted with teachers showed that in the overwhelming majority of cases they, themselves, didn't know on an operational level what is involved in the processes they try to teach students and request them to perform. In other words, they don't know appropriate methods, as nobody taught those methods to them.

Not surprising, therefore, are the recent findings of the National Assessment of Educational Progress which were summarized by the New York Times as follows: "American students have some understanding of basic scientific facts and principles, but their ability to apply scientific knowledge, design an experiment or clearly explain their reasoning is "disappointing", according to the latest national test of science education" (The New York Times, May 4, 1997).

5 Two Meanings Of The Term "Method" In Science And Everyday Language

There exist many definitions of the term "method" in philosophical and scientific literature which often create confusion and hamper communication among scientists. To clarify the issue, let's look at the use of the term "method" in everyday language.

The semantic analysis of the linguistic uses of the word "method" shows that it has two meanings: (a) actions leading to solving problems or performing tasks (as in the phrase "A scientist discovered a method of diagnosing a disease"), and (b) instructions (prescriptions) pointing out the actions to be performed (as in the phrase "A mathematician formulated a method for solving a certain class of problems").

To distinguish the two meanings of the term "method" which signifies two different, although connected, phenomena we will designate a method as a system of actions as M_a, and a method as a system of instructions - a prescription - as M_p. These designations will be used when it may not be clear from the context in which sense a method is meant. We will, however, not use these designations when the meaning is clear from the context or when what is said about a method refers both to M_a and M_p.

Normally, in searching for ways to solve new problems or perform new tasks people first discover (find) actions that lead to the solution, then designate or describe them verbally, and then, subsequently, convert descriptions into prescriptions as to what everyone should do in order to achieve a specified goal (solve a problem, perform a task, etc.). In other words, people first discover M_a's and then convert them into M_p's.

5.1 What A "Method" Precisely Is

We define "method" as a structured system of instructions and/or actions for achieving some goal.

This definition delineates the following essential characteristics of a method:

1. A method is always a system of instructions and/or actions, not just a single one. Only in extreme cases does the system consists of just one single instruction or action.

2. A method is always a structured entity which consists of basic elements (instructions and/or actions) connected in a certain manner (for example, organized in a certain sequence or hierarchy).

3. A method is always a goal-oriented phenomenon which is geared to achieving some goal (to perform a task, to solve a problem, etc.). It is not by accident that when people speak about methods they often use the preposition for ("a method for..."), although in certain contexts it may be more accurate to use the preposition of ("a method of...").

In everyday language and science the notion of a method is often conveyed by a number of full or partial synonyms, such as "process", "procedure", "guide", "technique","strategy" and some others. The problem is that some of them (like "strategy") are much more polysemantic and ambiguous than the term "method".

5.2 Methods As Objective Social And Subjective Psychological Phenomena

Once concepts and propositions are developed in social practice and science, they are objectivized (materialized) in language. Objectivized, they become a social phenomenon which objectively exist in printed or electronic form accessible to people for learning and using. Once learned, they become subjective psychological phenomena which represent subjective counterparts to the objectivized societal phenomena.

Subjective methods may or may not conform to the objectivized methods.For example, explanations given by many people are often faulty (incorrect, superficial, inconsistent, etc.). Their subjective methods of explanation do not conform to the effective objectivized methods of explanation developed in science.

Hence, one of the important objectives of education consists in teaching methods in such a way that the subjective methods formed in students' minds conform to the effective objectivized methods developed in social practice and science.

5.3 The Relationships Between The Notion Of A Method And The Notion Of A Skill

Although partially related, these notions are not the same, for they reflect different psychological phenomena.

As far as M_p's are concerned, it is obvious that the knowledge of actions to be performed in order to achieve some goal is not the same as the actual execution of actions. One can know, for example, how to swim (i.e., what actions to perform in order to swim) but not be able to swim (i.e., not be able to actually perform those actions). In this respect the notion of a method (M_p) is quite different from the notion of a skill.

A clear distinction must also be made between the notion of M_a as a system of actions and the notion of skills. Skills are not systems of actions, they are physiological processes in the brain which represent a potential for performing systems of actions, i.e., a potential for executing a method (M_a). This distinction becomes clear from the following simple example: When someone says about a surgeon that he is very skillful, the speaker does not mean that he is performing actions involved in surgery at this very moment.The meaning of this statement is that he can perform, he has the potential for executing those actions when the task of performance arises. Thus the notion of a method as a system of actions (M_a), not only as a system of instructions(M_p), is also different from the notions of skills.

Obviously, there is a direct connection between the systems of actions making up M_a's and skills: the performance of actions leads to the formation of physiological processes and associations in the brain which leave ”traces” after the actions cease to be executed. These ”traces” are skills. On the one hand, they are the results of the performed actions and, on the other hand, they represent a potential for their performance in the future, once actuated.

Another way to characterize skills is to say that they are actions (or systems of actions) in their latent form.

The fact that skills are formed only through performance of certain actions and their systems, makes it clear that in order to develop good, effective skills in students it is necessary to teach them good, effective methods. Teaching methods is a means for developing skills.

5.4 The Difference Between Knowledge Of And Command Of A Method

A clear distinction must be made between the knowledge of and the command of a method. To know a method means to know its instructions which manifests itself in the ability to verbally formulate them. To have command of a method means to be able to perform operations (physical and/or mental). The following situations may - and do - take place in schools and real life:

1. A person knows a method and has command of it.

2. A person knows a method but does not have command of it.

- *Example*: a person knows how to swim (has knowledge of the actions to be performed) but is unable to swim.

3. A person doesn't know a method but has command of it.

   - *Example*: a person is able to solve problems of a certain class by effectively performing a system of pertinent operations (making up M_a), but is unable to describe them or formulate a system of corresponding instructions.

4. A person neither knows a method nor has command of it.

6 Failure To Teach General Methods Of Thinking In Conventional Instruction And Its Negative Consequences

One of the problems encountered by practically all teachers is this: many students are able to solve problems similar to those they were taught to solve but are unable to solve problems of the same class or type which don't have enough outward similarity to the taught ones. Or they make errors in solving such problems. Why?

Our analysis has shown that it is because they were taught solution processes which, in the demonstrations provided, were applied to some selected content specific problems only - without teaching the general method of reasoning as a system of general enough instructions (M_p) as to what mental operations are to be used and applied to any content.

This kind of instruction is typical and widespread. As a result of such instruction, a system of actions (M_a) is associated in the students' minds only with the contents that were used in the demonstration. And only by such - and similar - contents the operations can be actuated in the process of solving other problems. That is why when confronted with problems, whose contents are dissimilar to those used in demonstrations, the students are either stuck(they don't know "what to do") or come to wrong solutions.

7 An Example of One of the Typical Methods Used in Conventional Instruction

Our former colleagues at the Institute of General and Educational Psychology in Moscow, Dr. Zykova, was present at a geometry lesson in one of the Moscow schools. The topic was ”right triangles”. This is what the teacher and the students were doing at the lesson:

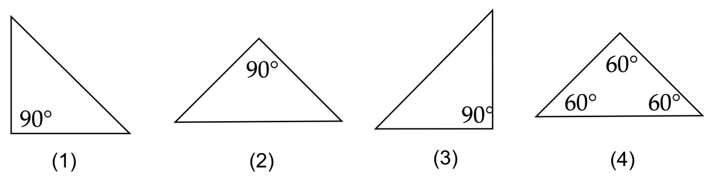
- **Pedagogical action 1**. The teacher explained that there are several types of triangles and that each type has specific characteristics. She said that today they would be studying right triangles and gave a definition (”a right triangle is a triangle which has a 90° angle”).
- **Pedagogical action 2**. She then demonstrated right triangles by giving several illustrations of this concept.
- **Pedagogical action 3**. She then provided practice for the students to learn both the concept (definition) of a right triangle and how to apply it:
  - **Action 3a**. She asked students to formulate the definition of a right triangle.
    - *A few students did it correctly.*
  - **Action 3b**. She then asked students to give examples of right triangles by drawing them on the blackboard.
    - *Two students did it correctly.*
  - **Action 3c**. Afterwards, she displayed several geometric figures on the blackboard, among which were right and non-right triangles, and asked students to identify the right triangles.
    - *Several students did it correctly.*
  - **Action 3d**. She then asked students if they had any questions or if everything was clear.

* *The students enthusiastically responded in chorus that everything was clear.*

Everything went very well and both the teacher and the students were sure that they had perfectly mastered the concept and learned how to apply it.

After the classes, Dr. Zykova asked one of the students, who was very active during the lesson and had correctly answered the teacher's questions, to participate in a small experiment. In it, she asked the student to give the definition of a right triangle. The student gave the correct definition saying that a right triangle is a triangle which has a 90° angle.

Then she offered him four triangles and asked him to indicate which of them were right triangles:



He chose (1) and (3). *Experimenter*: What about triangle (2)? It also has a right angle! Isn't it a right triangle?

Student: No, for a triangle to be right, the right angle should be at the bottom of the triangle either on the left or right hand side.

Despite the fact that the student gave the correct definition of a right triangle, this verbal definition was no more than a purely mechanical reproduction of the words spoken by the teacher when she formulated the definition. His actual concept of the right triangle was different from the correct concept of a right triangle contained in the teacher's definition and accurately verbally stated by the student. The correct concept did not include such irrelevant characteristic as the position of the right angle in a triangle. However, the student's concept included this irrelevant characteristic and, therefore, was incorrect (narrow).

How did the student's erroneous concept form in his mind when the instruction was seemingly well conducted (examples and counterexamples of the concepts were given, practice provided, etc.)? Despite the correct definition the student knew and gave, his actual concept reflected an empirical generalization of the objects (geometric figures) he encountered in the course of instruction, the spatial positions of the right angles in the triangles which were given as illustrations of the concept.

Zykova says that the major cause of the student's error was the limited variations of the spatial position of the right triangles provided in the illustrations. The students were shown triangles only in the "standard" positions (positions (1) and (3)). The pedagogical cure offered by Zykov (Zykova, 1963) and other representatives of the school of pedagogical thought advocating, consciously or unconsciously, empirical generalizations as the basis of learning and concept formation is: a teacher who introduces a new concept should vary - as widely as possible - the irrelevant features of the objects which illustrate the concept. This would prevent students from making limited generalizations and forming narrow concepts based on them.

Sure, limited variations of objects' irrelevant features is one of the causes of students' wrong concepts and errors in their decisions and solutions. But the major deep-rooted cause of the problem is, in our view, different.

7.1 Why It Is Impossible To Provide High Quality Instruction And Learning On The Basis Of Empirical Generalizations

Empirical generalizations, by varying irrelevant objects' characteristics, are good when the number of irrelevant variables and their related attributes is very small. But when it is greater than "very small", then the number of necessary variations gets so large that it becomes practically impossible to provide all of them in the course of instruction.(For example, if the number of irrelevant variables for a class of objects is only three (color, size, and spatial orientation), and each of them has only a few attributes, then the number of combinations of irrelevant characteristics will reach several dozen. No teacher is able to provide, in the course of instruction, several dozen variations of the same object, if only for the lack of time.)

If, however, the number of variations provided is less than objectively needed, there will always be the potential possibility that the generalizations in the students' minds will be incongruous with the scientific contents of the concepts. The discrepancy between the contents of the students' concepts and the contents of the scientific concepts is rather typical when learning any discipline in today's schools.

The greater the discrepancy between the objectively required number of variations and those actually provided, the greater the probability that (a) the generalizations formed in the students' minds will be inadequate and, as a result, (b) the rate of inaccurate concepts based on those generalizations will be very high.

The true cause of the problem of faulty concept formation and faulty concept application is, according to Landamatics, that students are not taught general methods of concept acquisition and application which does not require the impractical exhaustive variation of irrelevant features of objects. And if this is the true cause, then the preventive medicine and cure would be not the exhaustive variation of objects' irrelevant features but the teaching and learning of general methods of thinking. Only this can overcome the limitations of the empirical generalizations and form such very broad generalizations which are fully congruous with the scientific contents of the scientific concepts.

7.2 Landamatics Approach To Overcoming - Via Teaching Methods Of Thinking - The Limitations In Generalizations And In The Transfer Of Concepts And Mental Operations

Landamatics has developed and advocates a radically different approach to forming generalizations, concepts, and thought processes via purposeful and explicit teaching methods of thinking (both M_p's and M_a's).

- Makes a great number of variations unnecessary
- Guarantees the formation of proper, adequate generalizations

- Guarantees the formation, on the basis of adequate generalizations, of accurate concepts and propositions
- Guarantees the formation, within students, of effective methods of acquisition and application of knowledge (images, concepts and propositions)
- Guarantees the broadest and most accurate transfer, not limited by experience, of both knowledge and mental operations to new situations and problems
- Guarantees a dramatic reduction in errors and difficulties of learning
- Guarantees the development of the ability to self-manage, self-regulate, and self-control of one's own mental operations
- Makes it possible to achieve all of the above - reliably and relatively fast.

In contrast to the empirical generalizations formed in the minds of students who have had conventional instruction, the Landamatics approach forms **reliable, scientific concept-congruous (RSCC)** generalizations.

7.3 Design Of Instruction Based On The Landamatics Principle Of Teaching Methods Of Thinking

(an illustration)

To show, in a simple and contrasting way, the difference between the formation of a concept on the basis of an empirical generalization, on the one hand, and of the RSCC's generalization, on the other hand, we will use the same example of teaching students the concept of a right triangle, which would include teaching them a method of applying it to the task of recognizing (identifying) right triangles among (from?)other triangles.

As in teaching everything, two basic approaches (strategies, methods) can be used in teaching right triangles: (1) have students (get students to?) make an independent discovery of what is to be learned by properly guiding them, and (2) teach them ready-made knowledge and methods.

We will start with the first strategy (method).Here are the instructional objectives and activities which it involves:

1. The students' independent discovery of the concept of a right triangle. 2. Figuring out the triangle's name (the term used in science to designate the concept). 3. Framing the concept's logically correct definition. 4. The independent discovery of a system of mental operations (M_a) for applying the concept. 5. Formulation of the discovered method (M_p). 6. Learning, through practicing, how to apply the method. 7. Internalization of the method's instructions (M_p). 8. Automatization of the method's operations (M_a) and, thus, insuring its complete mastery and command.

7.3.1 Strategy (Method) 1

Guiding students towards making independent discoveries of a concept, its designation, its definition, and the method of applying it

Out of eight instructional objectives and activities listed above we will describe here the last five which are specific for the Landamatics method.

Instructional objective 4: Get students to discover and consciously realize the system of mental operations (M_a) involved in the application of the learned concept, and its definition, to the task of identifying objects as belonging or not belonging to the defined class (in this case, to the class of right triangles).

Pedagogical actions:

1. Ask the students what they should do in their heads in order to determine, on the basis of the definition, whether a triangle is a right triangle.

The students say that they have to check whether a triangle has a right angle.

Instructional objective 5: Get students to explicitly formulate the corresponding system of instructions (M_p).

Pedagogical actions:

1. Ask the students to formulate a detailed set of instructions, or commands - i.e., a method - of what a person, who does not know how to use the definition

of a right triangle, should do in his mind in order to determine whether some given triangle is a right triangle or not.

2. If the students formulate the method correctly, proceed to the next instructional objective; if not, then explain to them (the explanation is not given here for lack of space) how to formulate the method (M_p) of actions in order to recognize whether a triangle is right or not right.

Students formulate, with the teacher's help if needed, the following method:

1. Refer to the definition of a right triangle and isolate its characteristic feature - the presence of a 90 degree angle.

2. Mentally superimpose this feature on a given triangle and check to see if it has a 90° angle.

3. Draw a conclusion according to the following rules:

   - If a triangle has a 90° angle, then it is a right triangle.
   - If it does not have a 90° angle, it is not a right triangle.

4. Write down the formulated method (algorithm) on the blackboard or display it by any other medium (if prepared beforehand).

Instructional objective 6: Provide practice in the application of the formulated method (M_p).

Pedagogical actions:

1. Tell the students that the task now is to practice applying the formulated method for recognizing right triangles among other triangles.

2. Show them various triangles and have them determine, following the method, which of them are right triangles and which are not.

3. Explain that they should use the method in a step-by-step manner: look at the first instruction and do what it says, then look at the second instruction and do what it says, etc.

Following the method, the students easily identify right triangles regardless of the position of the right angle.

Instructional objective 7: Provide for the method's internalization, through special exercises, and thus insure its full mastery.

Pedagogical actions:

1. Tell the students that they seem to no longer need any more the instructions on the blackboard and seem to be able to replace them by self-instructions (self-commands).

2. Tell them that you will now erase the instructions on the blackboard and show a few more triangles. They should determine which of them are right triangles by giving themselves self-instructions as to what to do instead of following the instructions on the blackboard.

The students easily perform all the necessary mental actions (M_a) by giving themselves self-instructions.

Instructional objective 8: Effect automatization of the mental operations of the method (M_a).

Pedagogical actions:

1. Tell the students that they don't seem to further need even self-instructions, for they now know what to do in order to recognize a right triangle.

2. Show them the last set of triangles among which they have to find the right triangles. Ask them to find them as quickly as possible without giving themselves any self-instructions.

The students easily perform the assignment - find the right triangles instantaneously.

This completes the full circle of Landamatics-designed instruction based on Strategy 1.

Although the description of the Landamatics methodology of teaching and learning the concept of a right triangle and the method of its application was pretty long, in reality the entire lesson takes no more than 15-20 minutes.

7.3.2 Notes On The Psychological Mechanisms Of A Method's Internalization And Automatization Effected Via The Landamatics Instructional Methodology

What does it mean to internalize the instructions of the method (M_p) and automatize the operations of the method (M_a)? What happens in the mind during the processes of internalization and automatization?

According to the Landamatics theory, gradual internalization and automatization of a method is nothing more than a gradual shift, in the process of learning and practicing, from one kind of an operations' actuator to another.

1. At the first stage of learning a method, operations are actuated externally (from the outside) by the tangible method's instructions which exist in some tangible, material form (printed or electronic).

2. At the second stage, operations become actuated internally (from the inside) by the self-instructions. This is the stage of the method's (M_p's) internalization.

3. At the third stage, a need in any instructions (external or internal) disappears and the operations start to get actuated by the goals and problem conditions themselves. This stage is the stage of the operations' (M_a s') automatization.

In the course of moving from stage to stage, internal psychological mechanisms of mental processes undergo, according to Landamatics, one critical change: executed successively (in a step-by-step manner) at stages 1 and 2, mental operations start to be performed simultaneously (or partially simultaneously) at stage 3.

Simultanization of mental operations makes possible the following:

- Parallel processing of information instead of initial sequential processing
- Recognition of objects as patterns, as gestalts
- Carrying out mental operations (processes) very fast, instantaneously or almost instantaneously

- Carrying out mental operations (processes) without effort (they proceed as if by themselves).

These characteristics of mental processes are signs of their mastery and automatization.

In conventional instruction, these characteristics are formed (if formed) in a spontaneous, haphazard and often ineffective way. The Landamatics makes their formation a well-planned and instructionally well-managed process, thus guaranteeing the high quality of mental abilities developed as a result of simultanization.

7.3.3 Strategy (Method) 2

(Teaching concepts, terms, definitions, and methods in ready-made form)

With Strategy 2, instead of having the students discover the concept of a right triangle, figure out its term and frame its definition (as was the case with strategy 1), the teacher simply teaches all this knowledge to the students in ready-made form (with appropriate illustrations and exercises).

7.3.4 Conditions for Choosing Between Strategies 1 and 2

It is obvious that Strategy 1 is educationally more valuable,advantageous and beneficial than Strategy 2. But Strategy 1 takes more time.

It seems that the only condition for choosing one or another strategy is the amount of available time. Not infrequently, however, there is not enough time for using full-fledged discovery strategy 1 but more time available than needed for using Strategy 2. For this situation Landamatics suggests to use both strategies in a certain proportion. We call this a mixed, or combination, strategy.

7.3.5 Strategy (Method) 3

(combination strategy)

With this strategy, certain things within a topic are taught using the discovery strategy, and certain other things are taught by providing knowledge in ready-made form. Which topics should be taught by one or by the other strategy is determined by the teacher's objectives at the given moment and by the relative benefits that each of the method would provide with regard to each particular topic to be taught.

7.4 The Substantial Drawback Of The Method Of Thinking Taught In The Geometry Lesson Described Above

The method of thinking formulated for identifying right triangles was general in the sense that it could be applied to identification of any right triangle, but it was, at the same time, very specific, for it could be applied to identification of only the right triangles.

Is it possible to modify this method so that it will be applicable to other contents as well? In other words, is it possible to make it more general? The answer is yes.

7.5 Increasing The Degree Of Generality Of A Method

As an example and a departure point, we will use the method of identifying a right triangle formulated earlier. Let us designate the lowest degree of generality as d_1, the next (higher) degree of generality as d_2, and so on. Because the degree of generality of the method for identifying a right triangle is the lowest, this method will have index d_1. Each method with a lower degree of generality will be placed in the left column of a (the?) table. It will be juxtaposed side by side with a method having the next higher degree of generality which will be placed in the right column of the table. Those juxtapositions will make the comparison of the methods' degrees of generality easier. The differing elements in the contrasting methods will be delineated by using italics.

Generalization 1: from d_1 to d_2

Table 1

Method 1 (d_1)	**Method 2 (d_2)**

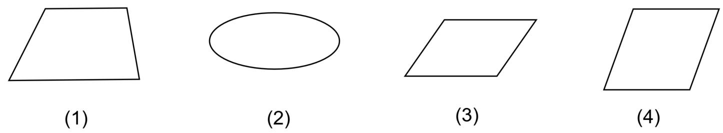
In order to identify a right triangle: 1. Refer to the definition of the right triangle and isolate its characteristic feature - the presence of a 90 degree angle. 2. Mentally superimpose this feature on any given triangle and check to see if it has a 90° angle. 3. Draw a conclusion according to the following rules: (a) If a triangle has a 90° angle, then it is a right triangle. (b) If it does not have a 90° angle, it is not a right triangle.	In order to identify an object as belonging or not belonging to a certain class: 1. Refer to the definition of the class and isolate its characteristic feature(s). 2. Mentally superimpose this feature(s) on any given object and check to see if it has all of the features. 3. Draw a conclusion according to the following rules: (a) If an object has all of the features, indicated in the definition, then it belongs to the class of objects defined in the definition. (b) If it does not have at least one of the features, it does not belong to this class of objects.

We suggest that the reader apply Method 2 - following its instructions in a step-by-step manner - to each of a number of geometric figures given below. The task is to determine (identify) which of them is a rhombus. Here is a definition of a rhombus which can be used: ”A rhombus is a parallelogram whose 4 sides have the same length”.

Here are a few figures, and we want to determine which one is a rhombus (if, of course, it is present here):

Now please apply this method to the task of identifying a right triangle (using examples of triangles given on p. 9). The reader will see that Method 2 is applicable to the identification of both a right triangle and a rhombus.

This means that it is more general than Method 1 which is applicable to identifying only a right triangle.



To appreciate the degree of generality of Method 2, we suggest that the reader apply it to solving a grammatical task of identifying a clause among the following groups of words: (a) ”My God!”; (b) ”Please, forgive me”; (c) ”When Peter entered the room”, and (d) ”I really like this book”. The following definition of a clause can be used: ”A clause is a group of words with a subject and a predicate” (Hirsch, 1993).

7.6 How General Is Method 2?

Browsing through a math course, we stumble upon the following rule: ”A number is divisible by fiVe if it ends in 5 or 0”. In order to test Method 2 for its degree of generality, let us select a few test numbers (for example, 15, 17, 20 and 23) and determine - following method 2 - whether they are divisible by 5. We suggest you go through this exercise. If you did, you should have come to a few erroneous conclusions.

Why? Obviously, it is because Method 2 is not general enough.

How do you find a more general method? In order to do so, it is necessary to find out why Method 2 worked successfully when applied to some concepts and definitions and didn’t work on some others.

7.7 Why Didn't Method 2 Always Work?

To diagnose the problem, let us compare the definitions of those concepts for which the method worked and the rule for which it didn't. How do they differ from each other? The difference is almost obvious: the characteristic features of both a rhombus and a clause are connected by the logical conjunction *and* (i.e., conjunctively), whereas the characteristic features of divisibility by 5 are connected by the logical conjunction *or* (disjunctively). Apparently, Method 2 works only for conjunctive concepts and propositions and does not work for disjunctive ones. The task now is to devise a method - Method 3 - for disjunctive structures of characteristic features.

Here is Method 3 as compared to Method 2:

Table 2

Method 2 (for conjunctive concepts) $\mathbf{d}_2$	**Method 3 (for disjunctive concepts) $\mathbf{d}_2$**
In order to identify an object as belonging or not belonging to a certain class: 1. Refer to the definition of the class and isolate its characteristic feature(s). 2. Mentally superimpose this feature(s) on any given object and check to see if it has all of the features. 3. Draw a conclusion according to the following rules: (a) If an object has all of the features, indicated in the definition, then it belongs to the class of objects defined in the definition. (b) If it does not have at least one of the features, it does not belong to this class of objects.	In order to identify an object as belonging or not belonging to a certain class: 1. Refer to the definition of the object and isolate its characteristic features. 2. Mentally superimpose this feature(s) on any given object and check to see if it has at least one of the features. 3. Draw a conclusion according to the following rules: (a) If an object has at least one of the features, indicated in the definition, then it belongs to the class of objects defined in the definition. (b) If it does not have all of the features (i.e., it has none of the features), it does not belong to that class of objects.

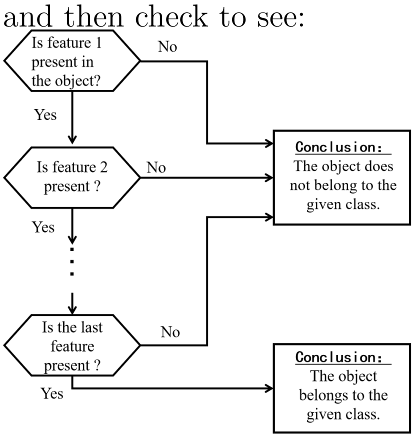
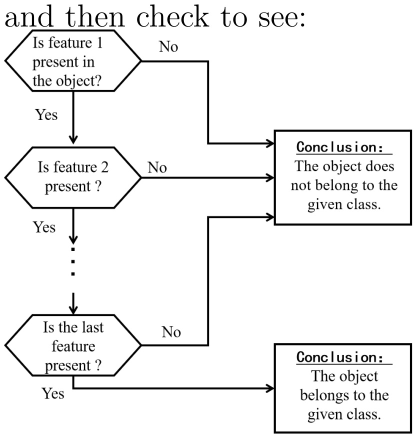
The reader can easily test Method 3 by finding definitions whose characteristic features are connected by the logical conjunction *or*. Here are some of them from various textbooks: "A change in the size or shape of something is a physical change"; "The indirect object answers the question, 'To whom?' or 'To what?'"; "Adjectives are the words we use to describe how something looks or feels or tastes or sounds".

Is Method 3 more general than Method 2? The answer is no. If Method 3 had subsumed Method 2, then it would have been more general than Method 2. But it does not, it just complements Method 2. This means that it has the same degree of generality.

7.8 A Graphic Representation Of General Methods

An effective way of describing, in detail, the system of actions involved in executing a method is to represent it graphically in flowchart form (see next page).[2]

Table 3

<table>
<tr><th>General method 2 for applying conjunctive concepts and identifying objects whose features are connected by the logical conjunction AND</th><th>General method 3 for applying disjunctive concepts and identifying objects whose features are connected by the logical conjunction OR</th></tr>
<tr><td>In order to identify whether some object belongs or does not belong to a certain class, refer to the class's definition, isolate its characteristic features and then check to see:

</td><td>In order to identify whether some object belongs or does not belong to a certain class, refer to the class's definition, isolate its characteristic features and then check to see:

</td></tr>
</table>

[2]Graphic representations organizing mental and other processes have been recently labeled "graphic organizers" (see, for example, S. Parks & Black, 1990, 1992).

7.9 Is A Combination Of Methods 2 And 3 General Enough To Handle Any Logical Structure Of Characteristic Features?

When testing Methods 2 and 3 for their generality, we have encountered the following two propositions:

1. A provision of the Social Security Administration: ”Only unmarried mothers who have one dependent child and whose income does not exceed $12,000, or married mothers who have two dependent children and whose household income does not exceed $16,500 are eligible for this benefit”.

2. A grammatical definition: ”A verb is a word that is characteristically the grammatical center of a predicate and expresses an act, occurrence, or mode of being” (Webster’s guide to business correspondence, 1988, p. 217).

How do you determine whether a specific woman is eligible for the benefit in question or not? How do you make the determination? How should you reason? Similarly, how do you determine whether a specific word is a verb or not? How should you reason?

Neither method 2, nor Method 3 is applicable to those propositions, for each of them contains both the conjunction *and* and the disjunction *or*. In other words, the logical structures of conditions of ”eligibility for this benefit” and characteristic features of a verb are neither purely conjunctive nor purely disjunctive. It is a mixed structure. But Method 2 and Method 3 are designed, each, for purely conjunctive or purely disjunctive logical structures and do not say anything about how to handle mixed structures. Therefore, they are not general enough and a more general method or methods are needed.

7.10 A Method For Discerning The Inner Logical Design Of Mixed Logical Structures Of Complex Concepts And Propositions

Let's demonstrate the method offered by Landamatics by example of the Social Security Administration provision cited in the previous section:

- **Operation 1.** Convert a proposition stated in the categorical form into the conditional if,...then form.

  "If an unmarried mother has one dependent child and her income does not exceed $12,000 or a married mother has two dependent children and her household income does not exceed $16,500, then, and only then, is she eligible for the benefit in question."

- **Operation 2.** To reveal the converted proposition's inner logical structure, describe it using parentheses.

  "If (an unmarried mother has one dependent child and her income does not exceed $12,000) OR (a married mother has two dependent children and her household income does not exceed $16,500), then, and only then, she is eligible for the benefit in question."

- **Operation 3.** Present the revealed logical structure in the graphic "logic diagram" form which makes the structure more transparent and distinct.

Logic diagram

IF

I.

(a) an unmarried mother has one dependent child
and
(b) her income does not exceed $12,000

OR

II.

(a) a married mother has two dependent children

and

(b) her household income does not exceed $17,500,

THEN, and only then, is she eligible for the benefit in question.

- **Operation 4.** Express the proposition (in its sentential or in logic diagram form) in a formula of propositional logic which describes its logical structure succinctly in the most generalized form.

Let us designate condition I(a) by letter a, condition I(b) by b, condition II(a) by c, condition II(b) by d, and the conclusion "she is eligible for the benefits in question" by E. We will further designate the logical conjunction **and** by &, the logical disjunction **or** by $\vee$, the if..., then connection in one direction as $\rightarrow$, and the if..., then connection in both directions as $\leftrightarrow$.

Then in the language of propositional logic our formula will look like this:

$$(a\&b) \vee (c\&d) \leftrightarrow E.$$

The formula reads as follows: If there are conditions a and b OR conditions c and d, then, and only then, draw conclusion E.

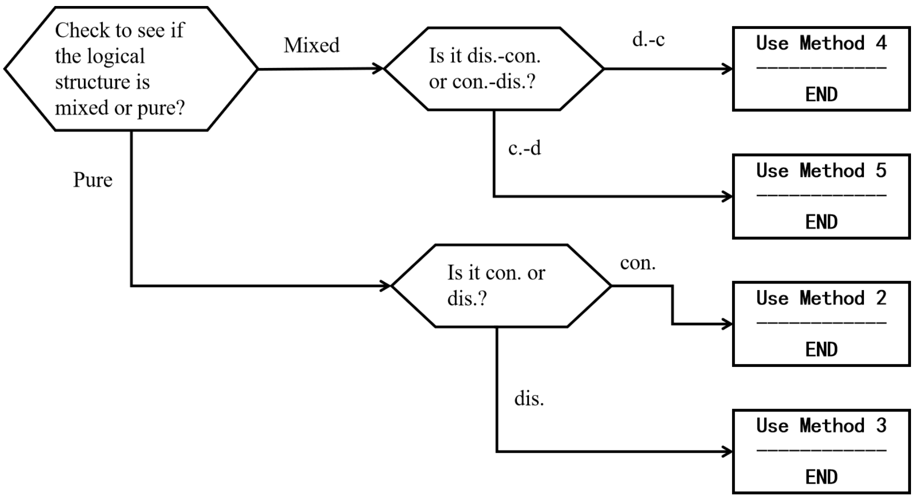
We suggest that the reader apply the just described method to the definition of a verb and then compare the logical structures of both propositions. Once done, it will become obvious that while the first proposition is a disjunction of conjunctions, the second proposition is a conjunction of disjunctions.

Now we suggest that the reader on his or her own figure out a method for applying the disjunctive-conjunctive concepts and propositions (Method 4) and a method for applying conjunctive-disjunctive propositions (Method 5). These methods will have a greater degree of generality (d_3) than methods 2 and 3 whose level of generality was d_2.

7.11 A Still More General Method Is Needed

Now we need a unifying more general method (d_4) which will subsume and bring together in a single system all the methods developed so far.

Here is how it can look:



7.12 Is Method 6, Finally, the Most General Method? Let Us See...

The methods formulated above were developed for the application of concepts with different logical structures of their characteristic features which were reflected in the concepts' definitions. But definitions are just one kind of propositions. Other kinds of propositions are rules, axioms, theorems, laws of nature (of physics, chemistry, biology, etc.), statements about attributes of objects and their relationships, and some others. Students encounter all of them in the studies at school and they are supposed to learn and know how to apply them.

In order for the formulated Method 6 to be the most general method for applying knowledge, it must work on any knowledge, including rules, laws and other kinds of propositions. Such a method must be applicable to all of them. Does method 6, and the less general methods on which it draws, work on all kinds of propositions? Let's test it.

Let's consider a simple geometric proposition (theorem) about one of the attributes of squares: "The diagonals of a square are perpendicular".

In the conditional if...,then form: "If a geometric figure is a square (S), then its diagonals are perpendicular (dp)".

In the language of propositional logic: $S \rightarrow dp$.

This is obviously a true statement. Now let us inverse it: "If the diagonals in a geometric figure are perpendicular, then this figure is a square: $dp \rightarrow S$. This statement is not true, for a figure that has perpendicular diagonals may also be a rhombus, not just a square.

Thus, statement $S \rightarrow dp$ in our example is true, but the inverse statement $dp \rightarrow S$ is not true.

There exist only one kind of propositions which are always true in both directions - the definitions.Other propositions that are true in one direction may or may not be true in the other direction. Their truth or falsity in the other direction must be determined in each particular instance.

The methods described so far were general only with regard to definitions and those other two-directional propositions which are true in both directions. But these methods are not applicable - or not completely applicable - to one-directional propositions which are true only in one direction. This means that the described methods are still not most general.

The methods for applying one-directional propositions are partially different from the methods of applying two-directional propositions. In the following table we will describe methods for the pure conjunctive (Method 2a) and the pure disjunctive (Method 3a) structures within the one-directional propositions. We suggest that you compare them with their corresponding methods 2 and 3 (p. 19) designed for two-directional propositions to see the difference. Obviously, the need for Methods 2a and 3a creates a need for Methods 4a and 5a which the reader can easily create by modifying Methods 4 and 5.

7.13 Methods For Pure Conjunctive And Disjunctive Structures Within One-Directional Propositional Knowledge

Here are the methods:

Table 4

Method 2a(for conjunctive concepts and conditions expressed in one-directional propositions) d_2	**Method 3a (for disconjunctive concepts and conditions expressed in one-directional propositions)** d_2
In order to identify an object as belonging or not belonging to a certain class or to determine whether to perform an action indicated in the right part of an if..., then proposition: 1. Refer to the proposition and isolate the characteristic feature(s) or conditions indicated in its left part. 2. Mentally superimpose the feature(s) or condition(s) on any given object or situation and check to see if it has the feature(s) or the condition(s). 3. Draw a conclusion according to the following rules: (a) If an object or situation has all the features or conditions, indicated in the left part of the proposition, then it belongs to the class of objects specified in the proposition's right part. If the proposition's right part indicates an action to be performed, this is the action to execute. (b) If an object or situation does not have at least one of the features or conditions, then no conclusion can be drawn. If an action is indicated in the right part of the proposition, it is not known whether this is the action to be performed.	In order to identify an object as belonging or not belonging to a certain class or to determine whether to perform an action indicated in the right part of an if..., then proposition: 1. Refer to the proposition and isolate the characteristic feature(s) or conditions indicated in its left part. 2. Mentally superimpose this feature(s) or condition(s) on any given object or situation and check to see if it has the feature(s) or the see if it has the feature(s) or the condition(s). 3. Draw a conclusion according to the following rules: (a) If the object or situation has at least one of the features of conditions, indicated in the left part of the proposition, then it belongs to the class of objects specified in the proposition's right part. If the proposition's right part indicates an action to execute (b) If the object or situations does not have all of the features or conditions, then no conclusion can be drawn.If an action is indicated in the right part of the proposition, it is not known whether this is the action to be performed.

An example to instruction 3b of Method 2a. Let us suppose that someone has formulated the following if...,then rule with an action indicated in its right part: ”If it is raining, take an umbrella when leaving the home”. Now suppose that it is not raining. Must I not perform the action, i.e., not take an umbrella? Not necessarily. I still may take the umbrella if I expect rain later in the day. The rule says what to do if the condition is present but does not say what to do if it is not present. The rule offers no conclusion about what to do if it is not raining. It leaves the decision open.

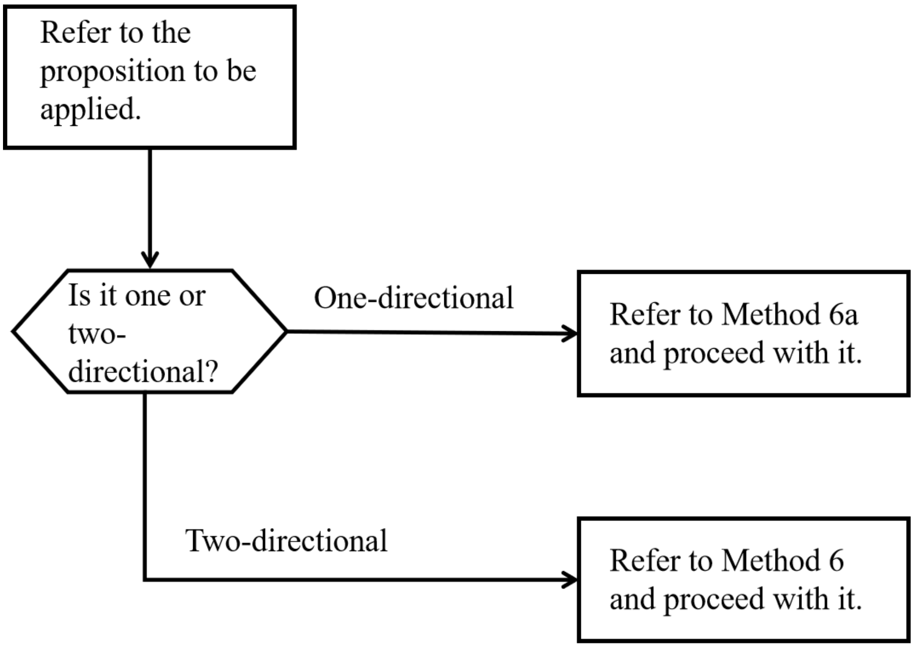
An example to instruction 3b of Method 3a. Let us suppose that someone has formulated the following if...,then rule with an action indicated in its right part: ”If it is raining or you expect it to rain later in the day, take an umbrella when leaving home”. Now suppose that it is not raining nor is expected to rain. Must I not perform the action, i.e., not take an umbrella with me? Not necessarily. I still may take the umbrella for some other reason. The rule says what to do if at least one of the conditions is present but it does not say what to do if neither of them is present. It leaves the decision open.

It is obvious that the modifications which were made in Methods 2 and 3 to turn them into Methods 2a and 3a (to fit one-directional propositions) should be made in all the other methods (4 through 6) because they are also based on Methods 2a and 3a. We suggest that the readers make these modifications and develop Methods 4a through 6a on their own.

7.14 Method 7 (the most general)(d_5)

Now we have arrived - at last! - at the most general method for learning and applying any conceptual knowledge expressed in any kind of propositions (definitions, rules, theorems, laws, etc.). These propositions may have any logical structure of characteristic features or conditions in the left part of the if...,then statement and they may have any of the two kinds of connections between the left and the right parts (two-directional or one-directional).

Here are the instructions which should precede methods 6 and 6a in order to make method 7 the most general:



Note. In propositions covered by the methods described in this chapter, the connection between the antecedent and the consequent is deterministic.There are also propositions which have a probabilistic connection where the consequent follows from the antecedent with some probability rather than certainty. That is why Method 7 is the most general with respect to deterministic propositions only.

7.14.1 How Difficult Is It To Teach Students The Most General Method Of Applying Conceptual And Propositional Knowledge?

From the description of the most general Method 7, and the way we have arrived at it, an impression may have arisen that teaching and learning it is a difficult and lengthy process. In fact, it is an easy and relatively fast process which students greatly enjoy. Moreover, it is possible to teach even junior high school students how to independently discover both the basic logical structures of concepts and propositions and the methods of handling them. In chapter 15 of (Landa, 1974), a detailed lesson is described. We strongly suggest that the readers familiarize themselves with that lesson, as it almost gives a script on how to conduct it. The methodology for conducting the lesson is so precise and well structured that it almost represents a teaching (instructional) algorithm which any teacher can follow and use.

After the original Russian 1966 edition of the book (Landa, 1974) was published, some preliminary experiments were conducted with primary school students. They showed that younger children, too, can be taught the basic logical structures of concepts and proposition, and methods of handling them.

It takes more time to teach students some auxiliary methods which are discussed in the next section.

7.15 The Explicit And Implicit Logical Structures Of Propositions. Why Auxiliary Methods Need To Be Taught And Learned To Make The General Method Work

In the majority of examples which we used to build the most general method of applying knowledge, the logical conjunctions and, or, not and if...,then were present in the propositions, and, thus, were explicit. This made discerning the logical structures and their representation in the logic diagram form relatively easy. However, in many propositions, both in science and everyday language, the logical conjunctions are expressed by grammatical conjunctions or are not present at all, which makes them and their related logical structures hidden or implicit.

Landamatics has developed methods for ”explicitating” hidden or implicit logical structures which cannot be discussed here for lack of space.We will limit ourselves to a few examples just to give an idea of the nature of the problem.

Examples translation of grammatical conjunctions into logical conjunctions

Grammatical conjunctions	**Translation into logical conjunctions**
1. He is smart but lazy. 2. I will not forgive him unless he apologizes. 3. The bank robber said to the tellers, ”Keep quiet and you will be OK”.	1. He is smart and lazy. 2. If he does not apologize, then I will not forgive him. Also: If he apologizes, then I will forgive him. 3. The bank robber said to the tellers, ”If you keep quiet, then you will be OK”.

We call the translation of grammatical conjunctions into the logical conjunctions and, or, not, and if...,then the reduction of propositions to their standard logical form, or logical standardization. Only a reduction of this kind clearly brings to light the logical structures of characteristic features and conditions, and makes it possible for a person to correctly and effectively use the methods of applying knowledge described above.

We suggest that the reader select or make up a number of sentences with various grammatical conjunctions (for example, therefore, rather than, otherwise, neither...nor, provided and others) and translate them into standard logical form.

Example of explicitation of an implicit logical structure

Implicit logical structure	**Explicitated logical structure**
1. ”A participial phrase is a group of related words containing a participle.” (Warriner & Griffith, 1957, p. 37)	1. If, and only if, the words in a group are (a) related and (b) contain a participle, then this group of words is a participial phrase.

We suggest that the reader select a number of propositions with implicit logical structures and then explicitate them. An example of such a proposition may be the following definition: ”An adverb is a word or a combination of words typically serving as a modifier of a verb, an adjective, another adverb, a preposition, a phrase, a clause, or a sentence and expressing some relation of manner or quality, place, time, degree, number, cause, opposition, affirmation, or denial" (Webster's Guide to Business Correspondence, 1988, p. 197).

8 The Educational Value Of Discovering, Teaching And Learning General Methods Of Thinking

Teaching and learning general methods of thinking have the following important educational benefits:

- It equips students with uniform and ubiquitous tools to acquire, manipulate, and apply knowledge of any content across all disciplines.

- It requires teaching and learning of each of the methods just once, making it unnecessary to teach and learn how to acquire, manipulate, and apply each particular knowledge.

- It saves an enormous amount of time, and thus vastly increases the productivity of both teaching and learning.

- It enormously increases the quality of acquired knowledge, skills, and abilities.

- It dramatically reduces difficulties in teaching and learning.

- It prevents many errors or immensely reduces their rate.

- It creates expert-level learners and performers almost from everyone and does this reliably and relatively fast.

8.1 Some Additional Educational Benefits Derived From Teaching And Learning General Methods Of Thinking

Here are some additional, but extremely important, educational benefits derived from teaching general methods of thinking:

- Students begin to understand the general makeup and structure of knowledge - any knowledge - regardless of its specific domain and contents, which leads to the development of interdisciplinary thinking.
- Students acquire a powerful tool for structural analysis and comparison of knowledge regardless of its contents and domain specificity.
- They acquire a tool and develop the ability to see the common (general) in the particular (specific).
- They begin to easier transfer knowledge, mental operations and their systems (general and more specific methods) from one content to another both within the same subject matter and between different subject matter domains; the range of transfer becomes incomparably broader.
- They become conscious of their own thinking processes and acquire the tool, and the ability, to self-manage, self-regulate, and self-control these processes. Their thinking becomes truly self-sufficient and independent.
- They develop general approaches to attacking different problems within the same or different domains of knowledge.

8.2 Why Are General Methods Of Thinking Not Commonly Taught In Schools Today?

There are several reason for it:

1. The insufficient maturity of educational science which has yet to realize the critical importance of teaching students of all ages general methods of thinking.

2. The underdevelopment of general methods of thinking in pedagogy and psychology, which results in a lack of scientific knowledge of the makeup and structure of different methods of thinking.

3. The underdevelopment in pedagogy and psychology of instructional methods for teaching general methods of thinking.

4. The focus, in instruction practice, on teaching and learning specific knowledges and skills rather than general methods of knowledge acquisition, manipulation, and application, on whose basis, from Landamatics' point of view, specific knowledges and skills should be taught and learned.

5. The unawareness or insufficient awareness that most teachers have - and that many professionals and expert performers in all areas of activity have - of their own mental processes and methods of thinking, which makes the communication of these methods and their transfer to students practically impossible.

6. The flaws in teacher preparation and training result in the fact that student teachers and practicing teachers do not learn either general methods of thinking (and other methods of cognitive activity) or general methods of teaching general methods of thinking.

8.3 A Brief Summary Of Problems In Learning And Thinking Resulting From Not Teaching Students General Methods Of Thinking

Here is a brief summary of problems in learning and thinking which develop when students are taught neither general methods of thinking nor how to discover them on their own: 1. If general methods of thinking are not taught, students are forced to try to discover them on their own. 2. If methods for the discovery of general methods of thinking are not taught, then students can use the only method available to them - trial and error. 3. Discovering general methods of thinking by trial and error is a difficult process (hence, the difficulties and problems in learning and thinking). 4. Discovering general methods of thinking by trial and error is a long process (hence, the duration of instruction and learning in each particular topic is too long). 5. Discovering general methods of thinking by trial and error is, as a rule, an unsystematic and haphazard process. 6. The discovered methods are, very often, based on empirical generalizations and are not general enough (they enable only limited transferability and limited areas of application). 7. Very often, not all the component actions are discovered and, as a result, the discovered methods are defective in one or several respects(incomplete, ineffective, etc.). 8. In cases where the discovered methods are correct and general enough, they are often inefficient (not economical). 9. Students who discover the operations of a method (Ma) through trial and error are, as a rule, unaware of them, for the operations don't reach the level of consciousness (Me). As a result, students are unable to self-manage, self-regulate, and self-control their mental processes. 10. Because of the unawareness of mental operations, students cannot communicate their mental processes and their systems (Ma's) to other people.

9 Are General Methods Of Thinking Content-Free?

The answer is yes if under content one understands the features that make, for example, a triangle different from a rhombus or a noun. But the answer is no if one includes in the notion of a content also the logical structures of those features. A logical structure of a content is also a content although of a radically different nature. Methods are not determined by the contents of the first kind but are determined and reflect the contents of the second kind.

The power of general methods of thinking consists in the fact that they allow one to isolate contents of the second kind and mentally separate them from the contents of the first kind. This makes it possible to apply mental operations to any content of the first kind, even such which was never encountered in the past experience. Thereby general methods of thinking enable people to overstep the limits of their past experiences and effectively think about things with which they had no prior personal experience.

10 General Methods Of Thinking And Intelligence

Finally, cognitive psychology in the USA and some other countries came to the thesis that intelligence is teachable and learnable (see, for example, Wimbey Wimbey, 1975; Sternberg, 1983; Perkins, 1995). (This thesis, incidentally, was put forward in Soviet cognitive and educational psychology several dozen years ago). What, however, is specifically teachable? What kind of processes or mechanisms? Until there is a clear and precise answer to this question, the thesis about teachability and leamability of intelligence hangs in the air. In order to know how to teach – produce – intelligence, it is necessary to know what precisely it is.

According to Landamatics, general intelligence is nothing other than a command (not knowledge or not just knowledge!) of a system of the most general methods of thinking applicable to any content-specific knowledge.

What does to teach and learn intelligence mean, then?

It means, according to Landamatics, to teach and learn general methods of thinking which lead to the development of general intelligence. One note is necessary here. Intelligence is not the performance of operations which make up methods (Ma's). Intelligence is what is left in the brain as a result of performing the methods' operations. Intelligence is (are?) the "traces" of previously performed systems of operations, their aftereffects.

This can be expressed in another way: intelligence cannot be taught or learned, only methods can. Intelligence can only be formed as a result of performing and internalizing methods' operations.

11 Limitatiions

We have been dealing, in this chapter, with only deterministic methods of knowledge application which are based on the full information about the objects to which knowledge is applied. There are, however, probabilistic general methods of cognitive activity and thinking that lie at the foundation of probabilistic intuitive judgments. The discussion of the probabilistic methods of cognitive activity and the instructional methods of teaching them is a separate topic.

12 References

Hirsch, Jr., E. D. (Ed.).(1993). What your 6th grader needs to know. Fundamentals of a good sixth-grade education (p.61). New York: Delta.

Lando., L.N. (1974). Algorithmization in leartning and instruction. Englewood Cliffs, NJ: Educational Technology Publications.

Landa, L. N. (1983). The algo-heuristic theory of instruction. In Ch. M. Reigeluth (Ed.), Instructional-design theories and models: An overview of their current status. Hillsdale, NJ: Lawrence Erlbaum Associates.

Landa, L. N. (1997). The algo-heuristic theory and methodology of learning, performance, and instruction as a paradigm. In Ch. R. Dills A.J. Romiszowski (Eds.), Instructional development paradigms. Englewood Cliffs, NJ: Educational Technology Publications.

Parks, S., Black, H. (1990, 1992). Organizing thinking. Graphic organizers. Books 1 and 2. Pacific Grove, Calif.: Critical thinking press software.

Perkins, D. (1995). Outsmarting IQ: The emerging science of learnable intelligence. New York: Free Press.

Sternberg, R. (1983). How can we teach intelligence. Research For Better Schools

Warriner, J. E. Griffith, F. (1957). English grammar and composition. New York: Harcourt, Brace World, Inc.

Webster's guide to business correspondence (1998). Mertriam-Webster Springfield, Mass.: Inc., Publishers.

Whimbey, A. Whimbey, L.S. (1975). Intelligence can be taught. York: E.P. Dutton.

Zykova, V. I. (1963). The formation of practical skills at geometry lessons (in Russian). Moscow: APS publishing house.